Proof or spoof ?	Season	2
	Episode	02
	Time frame	1 period

Objectives :
- Work on the concept of proof and discover some methods.

Materials :
- *Titles and explanations for some methods of proof.*

1 – Matching game — 25 mins

Students work in groups of 4 or 5. They are given the titles and explanations or examples for different kinds of proof, all mixed up. They have to put the title back with the right explanation or example.

2 – Find the valid ones — 30 mins

Each group has to find out which methods are valid and which ones are not.

Proof or spoof ?	Season	2
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	Document	Explanations

Type of proof		Valid	Not valid

An offshoot of Proof by Induction, one may assume the result is true. Therefore it is true.

Type of proof		Valid	Not valid

Although not a formal proof, a visual demonstration of a mathematical theorem is sometimes called a "proof without words". It is often used to prove the Pythagorean theorem.

Type of proof		Valid	Not valid

Also called proof by example, it is the exhibition of a concrete example with a property to show that something having that property exists. Joseph Liouville, for instance, proved the existence of transcendental numbers by constructing an explicit example.

Type of proof		Valid	Not valid

Multiply both expressions by zero, e.g.,

$$\begin{aligned} 1 &= 2 \\ 1 \times 0 &= 2 \times 0 \\ 0 &= 0 \end{aligned}$$

Since the final statement is true, so is the first.

Type of proof		Valid	Not valid

In this type of proof, the conclusion is established by logically combining the axioms, definitions, and earlier theorems. For example, it can be used to establish that the sum of two even integers is always even :

> Consider two even integers x and y. Since they are even, they can be written as $x = 2a$ and $y = 2b$ respectively for integers a and b. Then the sum $x + y = 2a + 2b = 2(a + b)$. From this it is clear $x + y$ has 2 as a factor and therefore is even, so the sum of any two even integers is even. This proof uses only definition of even integers and the distribution law.

Type of proof		Valid	Not valid

AN ARGUMENT MADE IN CAPITAL LETTERS IS CORRECT. THEREFORE, SIMPLY RESTATE THE PROPOSITION YOU ARE TRYING TO PROVE IN CAPITAL LETTERS, AND IT WILL BE CORRECT!!!!!1 (USE TYPOS AND EXCLAMATION MARKS FOR ESPECIALLY DIFFICULT PROOFS)

Type of proof		Valid	Not valid

By writing what seems to be an extensive proof and then smearing chocolate to stain the most crucial parts, the reader will assume that the proof is correct so as not to appear to be a fool.

Type of proof		Valid	Not valid

In this important type of proof first a "base case" is proved, and then an "inductive rule" is used to prove a (often infinite) series of other cases. Since the base case is true, the infinity of other cases must also be true, even if all of them cannot be proved directly because of their infinite number.
Its principle states that : Let $\mathbf{N} = \{1, 2, 3, 4, \ldots\}$ be the set of natural numbers and $P(n)$ be a mathematical statement involving the natural number n belonging to $\mathbf{N}$ such that
(i) $P(1)$ is true, i.e., $P(n)$ is true for $n = 1$;
(ii) $P(n+1)$ is true whenever $P(n)$ is true, i.e., $P(n)$ is true implies that $P(n+1)$ is true.
Then $P(n)$ is true for all natural numbers n.

Type of proof		Valid	Not valid

Remember, something is not true when its proof has been verified, it is true as long as it has not been disproved. For this reason, the best strategy is to limit as much as possible the number of people with the needed competence to understand your proof.
Be sure to include very complex elements in your proof. Infinite numbers of dimensions, hypercomplex numbers, indeterminate forms, graphs, references to very old books/movies/bands that almost nobody knows, quantum physics, modal logic, and chess opening theory are to be included in the thesis. Make sentences in Latin, Ancient Greek, Sanskrit, Ithkuil, and invent languages.
Again, the goal : nobody must understand, and this way, nobody can disprove you.

Type of proof		Valid	Not valid

In this type of proof, (also known as reductio ad absurdum, Latin for "by reduction toward the absurd"), it is shown that if some statement were so, a logical contradiction would occur, hence the statement must be not so. This method is one of the most prevalent of mathematical proofs. A famous example shows that $\sqrt{2}$ is an irrational number :

> Suppose that $\sqrt{2}$ is a rational number, so $\sqrt{2} = \frac{a}{b}$ where a and b are non-zero integers with no common factor (definition of a rational number). Thus, $b\sqrt{2} = a$. Squaring both sides yields $2b^2 = a^2$. Since 2 divides the left hand side, 2 must also divide the right hand side (as they are equal and both integers). So a^2 is even, which implies that a must also be even. So we can write $a = 2c$, where c is also an integer. Substitution into the original equation yields $2b^2 = (2c)^2 = 4c^2$. Dividing both sides by 2 yields $b^2 = 2c^2$. But then, by the same argument as before, 2 divides b^2, so b must be even. However, if a and b are both even, they share a factor, namely 2. This contradicts our assumption, so we are forced to conclude that $\sqrt{2}$ is an irrational number.

Type of proof		Valid	Not valid

"I believe assertion A to hold, therefore it does. Q.E.D."

Type of proof		Valid	Not valid

If enough people believe something to be true, then it must be so. For even more emphatic proof, one can use the similar Proof by a Broad Consensus.
Either kind of proof can be combined with other types of proof (such as Proof by Repetition and Proof by Intimidation ; e.g., "A Broad Consensus of Scientists ...") when required.

Type of proof		Valid	Not valid

It establishes the conclusion "if p then q" by proving the equivalent contrapositive statement "if not q then not p".

Type of proof		Valid	Not valid

The proposition is true due to the lack of a counterexample. For when you know you are right and that you don't give a shit about what others may think of you.

Type of proof		Valid	Not valid

In this type of proof, the conclusion is established by dividing it into a finite number of cases and proving each one separately. The number of cases sometimes can become very large. For example, the first proof of the four color theorem was a proof by exhaustion with 1,936 cases. This proof was controversial because the majority of the cases were checked by a computer program, not by hand. The shortest known proof of the four colour theorem today still has over 600 cases.

Type of proof		Valid	Not valid

Make it easier on yourself by leaving it up to the reader. After all, if you can figure it out, surely they can. Examples :

- "The reader may easily supply the details."
- "The other 253 cases are analogous."
- "The proof is left as an exercise for the reader."
- "The proof is left as an exercise for the marker." (Guaranteed to work in an exam.)

Type of proof		Valid	Not valid

Since August is such a good time of year, then no-one will disagree with a proof published then, and therefore it is true. Of course the converse is also true, i.e. January is shite, and all the logic in the world will not prove your statement.

Type of proof		Valid	Not valid

Reducing problems to diagrams with lots of arrows. This is related to proof by complexity.

Type of proof		Valid	Not valid

If there is a consensus on a topic. and you disagree, then you are right because people are stupid. See global warming sceptics, creationist, tobacco companies, etc., for application of this proof.

Type of proof		Valid	Not valid

Be sure to provide some distraction while you go on with your proof, e.g., some third-party announces, a fire alarm (a fake one would do, too) or the end of the universe. You could also exclaim, "Look ! A distraction !", meanwhile pointing towards the nearest brick wall. Be sure to wipe the blackboard before the distraction is presumably over so you have the whole board for your final conclusion.
Don't be intimidated if the distraction takes longer than planned and simply head over to the next proof.
An example is given below.

1. Look behind you !
2. ... and proves the existence of an answer for $2 + 2$.
3. Look ! A three-headed monkey over there !
4. ... leaves 5 as the only result of $2 + 2$.
5. Therefore $2 + 2 = 5$. Q.E.D.

Type of proof		Valid	Not valid

Suppose $P(n)$ is a statement.

1. Prove true for $P(1)$.
2. Prove true for $P(2)$.
3. Prove true for $P(3)$.
4. Therefore $P(n)$ is true for all n.

Type of proof		Valid	Not valid

If Jack Bauer says something is true, then it is. No ifs, ands, or buts about it. End of discussion.

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If you say something is true enough times, then it is true. If you say something is true enough times, then it is true. If you say something is true enough times, then it is true. If you say something is true enough times, then it is true. If you say something is true enough times, then it is true. If you say something is true enough times, then it is true.
Exactly how many times one needs to repeat the statement for it to be true is debated widely in academic circles. Generally, the point is reached when those around die through boredom.
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If you prove your claim for one case, and make sure to restrict yourself to this one, you thus avoid any case that could compromise you. You can hope that people won't notice the omission.
Example : Prove the four-color theorem. Take a map of only one region. Only 1 color is needed to color it and 1 is less than 4. End of the proof.
If someone questions the completeness of the proof, others methods of proofs can be used.

- Visual proof
- Proof by Multiplicative Identity
- Proof by Restriction
- Proof by Assumption
- Proof by Belief
- Proof by Construction
- Proof by Cases
- Proof by Complexity
- Proof by (a Broad) Consensus
- Proof by August
- Proof by Default
- Proof by Transposition
- Proof by Delegation
- Direct proof
- Proof by Chocolate
- Proof by Dissent
- Proof by Distraction
- Proof by Exhaustion
- Proof by Engineer's Induction
- Proof by Diagram
- Proof by Mathematical Induction
- Proof by Jack Bauer
- Proof by Repetition
- Proof by contradiction

Document 1 Methods of proof : titles and explanations

Direct proof

In this type of proof, the conclusion is established by logically combining the axioms, definitions, and earlier theorems. For example, it can be used to establish that the sum of two even integers is always even :

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Proof by mathematical induction

In this important type of proof first a "base case" is proved, and then an "inductive rule" is used to prove a (often infinite) series of other cases. Since the base case is true, the infinity of other cases must also be true, even if all of them cannot be proved directly because of their infinite number.
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Proof by transposition

It establishes the conclusion "if p then q" by proving the equivalent contrapositive statement "if not q then not p".

Proof by contradiction

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Proof by construction

Also called proof by example, it is the exhibition of a concrete example with a property to show that something having that property exists. Joseph Liouville, for instance, proved the existence of transcendental numbers by constructing an explicit example.

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