Formal Logic	Season	2
	Episode	03
	Time frame	1 period

Objectives :

- Discover the five logical operators "not", "and", "or", "if" and "iff".
- Learn how to prove that two logical propositions are formally equivalent.

Materials :

- *Fact sheet about formal logic.*
- *Beamer about formal logic.*

1 – Group work : the five operators — 20 mins

Students work in 9 groups of 4 or 5. They are handed out cards with a proposition involving a logical operator, or an explanation, the notation or a diagram. The have to match each proposition with the right explanation and notation. The answers are then given with a beamer.

2 – Lecture — 35 mins

The teacher presents the notions of truth table, negation, conjuntion, disjunction, conditional and biconditional. The lecture is based on a beamer, with hand-outs to be given at the end. Students must be involved in the lesson and help build the truth tables.

Formal Logic	Season	2
	Episode	03
	Document	Lesson

Formal logic is a branch of mathematics (and philosophy) where the validity of logical deductions and propositions is studied, without any reference to the truth of the statements involved. We only study the form, not the content.

1 A logical proposition and its negation

Any proposition, such as "it's raining" or "ABCD is a rectangle" can be true or false. For any proposition p, we will write 0 if it's false and 1 if it's true. The *truth table* of the proposition p gives all the possibilities about the truth of p. It's fairly simple when we consider just one proposition.

Definition 1 Negation

The *negation* of a proposition p, noted by the logical operator $\neg$ and the word *not*, is true when p is false. In other words, $\neg p$ is the contrary of p.

p	$\neg p$
0	1
1	0

Example : The negation of the proposition "the door is closed" is "the door is not closed".

2 The main logical operators

Using two or more propositions, we can build a new one. For example, "it's raining and I don't have an umbrella" is made of two distinct propositions. We will now consider some logical formulas made of two or more propositions.

Definition 2 Conjunction

The *conjunction* of two propositions p and q, noted by the logical operator $\wedge$ and the word *and*, is true when both propositions are true. In other words, $p \wedge q$ is true when p and q are true.

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Example : The conjunction of the propositions "I like chocolate" and "there's some chocolate in the kitchen" is "I like chocolate and there's some chocolate in the kitchen".

Definition 3 Disjunction

The *disjunction* of two propositions p and q, noted by the logical operator $\vee$ and the word *or*, is true when at least one of the propositions is true. In other words, $p \vee q$ is true when at least p or q is true. Beware, the logical disjunction is true when both propositions are true. It's different from the exclusive disjunction, often used in the common langage, as in "cheese or dessert", "open or closed".

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Example : The disjunction of the propositions "$ABCD$ is a rectangle" and "$ABCD$ is a diamond" is "$ABCD$ is a rectangle or a diamond". Note that both propositions may be true at the same time.

Definition 4 Conditional

The *conditional* of p and q, noted by the logical operator $\to$ and the words *if ... then ...*, is always true except if p is true and q is false.

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

Example : The conditional of "it's raining" and "I'll take my umbrella" is "if it's raining then I'll take my umbrella".

Definition 5 Biconditional

The *biconditional* of p and q, noted by the logical operator $\leftrightarrow$ and the words *if and only if*, is true when p and q have the same truth value.

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Example : The biconditional of propositions "ABC is a right angled triangle in A" and "$BC^2 = AB^2 + AC^2$" is "ABC is a right angled triangle in A if and only if $BC^2 = AB^2 + AC^2$".

3 Formal equivalences

Definition 6 Formal equivalence

Two logical statements are formally equivalent if they share the same truth table. To prove the formal equivalence of two statements, the easiest way is to compute their truth tables and compare them.

Proposition 1 Converse

Let p and q be any two propositions. The proposition $q \to p$ is called the converse of $p \to q$. It's not formally equivalent to $p \to q$.

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

Proposition 2 Contrapositive

Let p and q be any two propositions. The proposition $\neg q \to \neg p$ is called the contrapositive of $p \to q$. It's formally equivalent to $p \to q$.

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

Definition 7 Tautology and contradiction

A *tautology* is a logical statement that is true by virtue of its logical form, that requires no assumptions to determine its veracity. A *contradiction* is a statement that is always false.

Document 1 Matching game

Five sentences

1. a is not even.

2. a is even and b is divisible by 4.

3. a is even or b is divisible by 4.

4. If a is even then b is divisible by 4.

5. a is even if and only if b is divisible by 4.

Five notations

i. $(2|a) \vee (4|b)$

i. $(2|a) \to (4|b)$

iii. $(2|a) \leftrightarrow (4|b)$

iv. $\neg(2|a)$

v. $(2|a) \wedge (4|b)$

Five explanations

a. True only if both propositions are true.

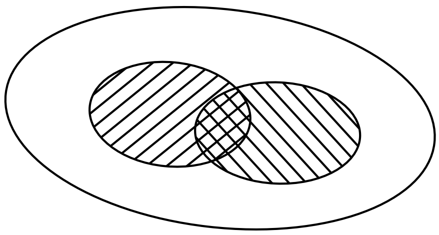
b. True if the proposition is false and false if the proposition is true.

c. True except if the second proposition can be false while the first property is true.

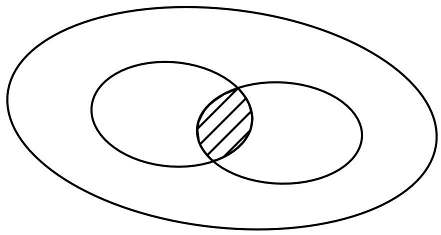
d. True as soon as one of the two propositions is true.

e. True if whenever one of the proposition is true, the other is also true, and whenever one proposition is false, the other is also false.

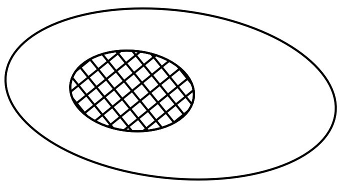
A



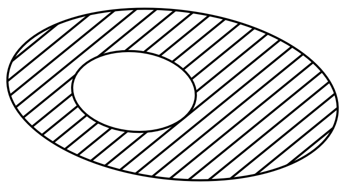
B



C



D



E