Episode 03 – Formal logic

European section – season 2

a is not even.

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- True if the proposition is false and false if the proposition is true.

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- True if the proposition is false and false if the proposition is true.
- $\neg(2|a)$

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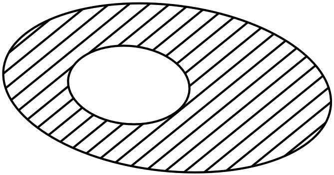

a is even and *b* is divisible by 4.

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- True only if both propositions are true.

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- $(2|a) \wedge (4|b)$

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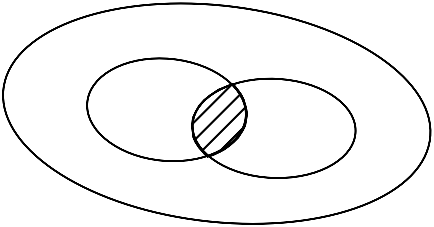

a is even or b is divisible by 4.

a is even or *b* is divisible by 4.

- True as soon as one of the two propositions is true.

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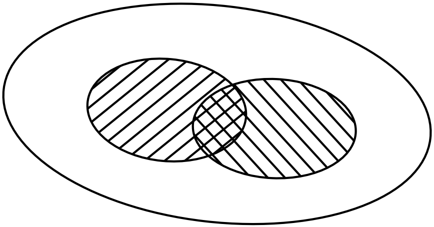
- $(2|a) \vee (4|b)$



If *a* is even then *b* is divisible by 4.

If a is even then b is divisible by 4.

- True except if the second proposition can be false while the first property is true.

If a is even then b is divisible by 4.

- True except if the second proposition can be false while the first property is true.
- $(2|a) \rightarrow (4|b)$

If a is even then b is divisible by 4.

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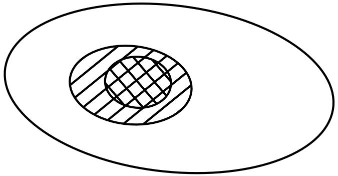
- $(2|a) \to (4|b)$



a is even if and only if *b* is divisible by 4.

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- True if whenever one of the proposition is true, the other is also true, and whenever one proposition is false, the other is also false.

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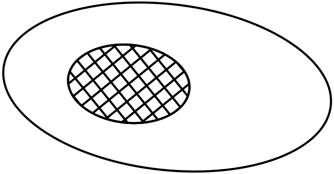
- $(2|a) \leftrightarrow (4|b)$



A logical proposition and its negation

Definition (Truth value)

For any proposition p, we will write 0 if it's false and 1 if it's true.

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p	$\neg q$
0	1
1	0

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p	$\neg q$
0	1
1	0

Example

The negation of the proposition "the door is closed" is "the door is not closed".

The main logical operators

Definition (Conjunction)

The *conjunction* of two propositions p and q, noted by the logical operator $\wedge$ and the word *and*, is true when both propositions are true. In other words, $p \wedge q$ is true when p and q are true.

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p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

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p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Example

The conjunction of the propositions “I like chocolate” and “there’s some chocolate in the kitchen” is “I like chocolate and there’s some chocolate in the kitchen”.

Definition (Disjunction)

The *disjunction* of two propositions p and q, noted by the logical operator $\vee$ and the word *and*, is true when at least one of the propositions is true. In other words, $p \vee q$ is true when at least p or q is true. Beware, the logical disjunction is true when both propositions are true. It's different from the exclusive disjunction, often used in the common langage, as in "cheese or dessert", "open or closed".

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p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
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p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Example

The disjunction of the propositions “*ABCD* is a rectangle” and “*ABCD* is a diamond” is “*ABCD* is a rectangle or a diamond”. Note that both propositions may be true at the same time.

Definition (Conditional)

The *conditional* of p and q, noted by the logical operator $\rightarrow$ and the words *if . . . then . . .* , is always true except if p is true and q is false.

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p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
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p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Example

The conditional of “it’s raining” and “I’ll take my umbrella” is “if it’s raining then I’ll take my umbrella”.

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0	0	1
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Example

The biconditional of propositions "ABC is a right angled triangle in A" and "$BC^2 = AB^2 + AC^2$" is "ABC is a right angled triangle in A if and only if $BC^2 = AB^2 + AC^2$".

Formal equivalences

Definition (Formal equivalence)

Two logical statements are formally equivalent if they share the one method is to build their truth tables and compare them.

Theorem

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Proof.

Let's look at the truth tables of the two propostions.

p	q	$p \rightarrow q$	$q \rightarrow p$
0	0	1	
0	1	1	
1	0	0	
1	1	1	

□

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Let p and q be any two propositions. The proposition $q \to p$ is called the converse of $p \to q$. It's not formally equivalent to $p \to q$.

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p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	
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1	0	0	1
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As the truth tables are not the same, the propositions are not formally equivalent. □

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Let p and q be any two propositions. The proposition $\neg q \to \neg p$ *is called the contrapositive of* $p \to q$*. It's formally equivalent to* $p \to q$*.*

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Let's look at the truth tables of the two propostions.

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1			
0	1	1			
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Theorem

Let p and q be any two propositions. The proposition $\neg q \rightarrow \neg p$ is called the contrapositive of $p \rightarrow q$. It's formally equivalent to $p \rightarrow q$.

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Let's look at the truth tables of the two propostions.

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
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1	0	0	1	0	0
1	1	1	0	0	1

As the truth tables are the same, the propositions are formally equivalent. □

Definition (Tautology and contradiction)

A *tautology* is a logical statement that is true by virtue of its logical form, that requires no assumptions to determine its veracity. A *contradiction* is a statement that is always false.