Episode 04 – Formal equivalences

European section – Season 2

De Morgan's laws

Theorem

$$\neg(p \land q) \iff (\neg p \lor \neg q)$$

De Morgan's laws

Theorem

$$\neg(p \land q) \iff (\neg p \lor \neg q)$$

Proof.

Let's look at the truth table of each proposition.

p	q	$p \land q$	$\neg(p \land q)$	$\neg p$	$\neg q$	$\neg p \lor \neg q$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

De Morgan's laws

Theorem

$$\neg(p \wedge q) \iff (\neg p \vee \neg q)$$

Proof.

Let's look at the truth table of each proposition.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

As $\neg(p \wedge q)$ and $\neg p \vee \neg q$ have the same truth table, they are formally equivalent. □

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1	0	1	0	0	1	0
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Conditional

Theorem

$$(p \to q) \iff (\neg q \to \neg p) \iff (\neg p \lor q)$$

Conditional

Theorem

$$(p \to q) \iff (\neg q \to \neg p) \iff (\neg p \lor q)$$

Proof.

The first formal equivalence has been proven in the previous session, so we just have to look at the truth tables of $p \to q$ and $\neg p \lor q$.

p	q	$p \to q$	$\neg p$	$\neg p \lor q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

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Theorem

$$(p \to q) \iff (\neg q \to \neg p) \iff (\neg p \vee q)$$

Proof.

The first formal equivalence has been proven in the previous session, so we just have to look at the truth tables of $p \to q$ and $\neg p \vee q$.

p	q	$p \to q$	$\neg p$	$\neg p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

As $p \to q$ and $\neg p \vee q$ have the same truth table, they are formally equivalent. □

Biconditional

Theorem

$$(p \leftrightarrow q) \iff (\neg q \leftrightarrow \neg p)$$

Biconditional

Theorem

$$(p \leftrightarrow q) \iff (\neg q \leftrightarrow \neg p)$$

Proof.

Once again, we just have to look at the truth tables of the two propositions.

p	q	$p \leftrightarrow q$	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	0	0	1

Biconditional

Theorem

$$(p \leftrightarrow q) \iff (\neg q \leftrightarrow \neg p)$$

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Once again, we just have to look at the truth tables of the two propositions.

p	q	$p \leftrightarrow q$	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	0	0	1

As $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ have the same truth table, they are formally equivalent. □