Set-builder notation	Season	2
	Episode	06
	Time frame	2 periods

Prerequisites : Logical operators, Venn diagrams and the concept of set

Objectives :

- Master the set-builder-notations.
- Build a glossary of useful symbols.
- Discover Russell's paradox

Materials :

- *Answer sheet : matching set definitions and lists.*
- *Glossary : symbols used to define sets.*
- *Beamer with the solutions, the glossary and Russel's paradox.*

1 – Matching sets and definitions in teams — 55 mins

Students work in groups of five. They are handed out a list of set definitions and lists of numbers that they have to match.

2 – Marking phase — 15 mins

While the answers are given by the teacher on a beamer, each team marks the answer sheet of another one.

3 – Glossary — 25 mins

Each team must build a glossary about sets. Theb, the main symbols used in the set-builder notation are shown and explained by the teacher.

4 – Russell's paradox — 15 mins

Russell's paradox is explained by the teacher.

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	Document	Answer sheet

Name : ____________________

Grade : ____________________

$\{2k+1 : k \in \mathbf{Z}\}$ ☐

$\{n : n \in \mathbf{Z} \wedge \dfrac{12}{n} \in \mathbf{Z}\}$ ☐

$\{n^2 : n \in \mathbf{N} \cap [0,5]\}$ ☐

$\{3k : k \in \mathbf{N}\}$ ☐

$\{x \in \mathbf{R} : 3x \in \mathbf{Z}\}$ ☐

$\{a + bi : a \in \mathbf{R} \wedge b \in \mathbf{R}\}$ ☐

$\{7k+1 : k \in \mathbf{Z}\}$ ☐

$\{n \in \mathbf{N} : \exists m \in \mathbf{Z}, m^2 = n\}$ ☐

$\{x \in \mathbf{R} : x^2 = x\}$ ☐

$\{x \in \mathbf{R} : \exists p, q \in \mathbf{Z}, q \neq 0 \wedge xq = p\}$ ☐

$\{r \in \mathbf{R} : \exists k \in \mathbf{Z}, r^2 = k\}$ ☐

$\{n \in \mathbf{Z} : |n| \leqslant 5\}$ ☐

$\{x \in \mathbf{R} : |x| \leqslant 5\}$ ☐

$\{x \in \mathbf{R} : \forall y \in \mathbf{N}, x \leqslant y\}$ ☐

$\{(x,y) : x \in \mathbf{N} \wedge y \in \mathbf{N} \wedge\ x + y = 7\}$ ☐

$\{x \in \mathbf{R} : x \leqslant -3 \vee\ x \geqslant 5\}$ ☐

$\{n \in \mathbf{Z} : \forall m \in \mathbf{Z}, n > m\}$ ☐

$\{n \in \mathbf{N} : \nexists m \in \mathbf{N} - \{1,n\}, \dfrac{n}{m} \in \mathbf{N}\}$ ☐

$\{(x,y) : x \in \mathbf{N} \wedge y \in \mathbf{N} \wedge\ xy \leqslant 6\}$ ☐

$\{(x,y) : x \in \mathbf{N} \wedge y \in \mathbf{N} \wedge\ y = 2x\}$ ☐

☐ $6,\ 21,\ 336,\ 4272$

☐ $3+5i,\ \sqrt{2}-i,\ 17,\ -1+\pi i$

☐ $-5,\ -2\pi,\ -\frac{4}{7},\ -0.0001$

☐ $0,\ 1,\ 9,\ 16$

☐ $-5,\ -\sqrt{19},\ 17.25,\ 5$

☐ $0,\ 9,\ 81,\ 196$

☐ $\frac{12}{7},\ -\frac{2}{3},\ 42,\ \frac{-28}{2}$

☐ $3,\ 6,\ -12,\ 1$

☐ $17,\ 5,\ 43,\ 199$

☐ $5,\ \sqrt{2},\ -13,\ \sqrt{13}$

☐ $(2,2)\,;\ (1,5)\,;\ (3,1)\,;\ (1,1)$

☐ $-4,\ -2,\ 0,\ 3$

☐ $\frac{7}{3},\ -17,\ -\frac{52}{3},\ 0$

☐ $0,\ 1$

☐ $-7,\ -3,\ 5,\ 17$

☐ $(3,6)\,;\ (12,24)\,;\ (5,10)\,;\ (131,262)$

☐ $(2,5)\,;\ (1,6)\,;\ (4,3)\,;\ (3,4)$

☐ $\sqrt{2},\ -\pi,\ 2.256,\ -4.99$

☐ $-20,\ 1,\ 50,\ 778$

☐ $\varnothing$

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Glossary

Set-builder notation

$\{\ldots\}$ **:** A set.

$:$ **or** $|$ **:** "such that".

Sets of numbers

$\mathbf{N}$ **:** The whole numbers (natural numbers and zero).

$\mathbf{Z}$ **:** The integers.

$\mathbf{D}$ **:** The decimal numbers.

$\mathbf{Q}$ **:** The rational numbers

$\mathbf{R}$ **:** The real numbers.

$\mathbf{R}^\star$ **:** The non-zero real numbers.

$\mathbf{R}^+$ **:** The positive real numbers.

$\mathbf{R}^-$ **:** The negative real numbers.

Set, subsets and elements

$\cap$ **:** The intersection of two sets.

$\cup$ **:** The union of two sets.

$\in$ **:** "is an element of the set"

$\subset$ **:** "is a subset of the set"

Logical operators

$\neg$ **:** The negation of a proposition, "not".

$\wedge$ **:** The conjunction of two propositions, "and".

$\vee$ **:** The disjunction of two propositions, "or".

$\forall$ **:** "For all"

$\exists$ **:** "There exists"

$\nexists$ **:** "There doesn't exist"

Russell's paradox

Let R be the set of all sets that don't contain themselves, that is

$$R = \{X : X \text{ is a set and } X \notin X\}.$$

Then R either is or is not an element of itself.

- If R is not an element of itself, then R is a set that doesn't contain itself, and so R is an element of R, which is precisely the contrary of what we supposed.
- Conversely, if R is an element of itself, then R does not contain itself, so we also get to a contradiction.

So R cannot be a set, or our definition of a set is not satisfactory.