Episode 06 – Set-builder notation

European section – Season 2

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$6, 21, 336, 4272$

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17, 5, 43, 199

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$(3, 6)$; $(12, 24)$; $(5, 10)$; $(131, 262)$

Set-builder notation

$\{\ldots\}$: A set.

: or $|$: "such that".

Sets of numbers

$\mathbf{N}$: The whole numbers (natural numbers and zero).

$\mathbf{Z}$: The integers.

$\mathbf{D}$: The decimal numbers.

$\mathbf{Q}$: The rational numbers

$\mathbf{R}$: The real numbers.

$\mathbf{R}^{\star}$: The non-zero real numbers.

$\mathbf{R}^{+}$: The positive real numbers.

$\mathbf{R}^{-}$: The negative real numbers.

Set, subsets and elements

$\cap$: The intersection of two sets.

$\cup$: The union of two sets.

$\in$: "is an element of the set"

$\subset$: "is a subset of the set"

Logical operators

- $\neg$: The negation of a proposition, “not”.
- $\wedge$: The conjunction of two propositions, “and”.
- $\vee$: The disjunction of two propositions, “or”.
- $\forall$: “For all”
- $\exists$: “There exists”
- $\nexists$: “There doesn’t exist”

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