Episode 17 – Cross sections and coordinates

European section – Season 2

Cartesian equations and planes

Definition

The set of points $M(x, y, z)$ in the 3D space such that

$$ax + by + cz = d$$

where a, b, c and d are real numbers is a *plane*.

Cartesian equations and planes

Definition

The set of points $M(x, y, z)$ in the 3D space such that

$$ax + by + cz = d$$

where a, b, c and d are real numbers is a *plane*.

Definition

The previous equality, true only for the points on the plane, is a *cartesian equation* of the plane.

Cartesian equations and planes

Definition

The set of points $M(x, y, z)$ in the 3D space such that

$$ax + by + cz = d$$

where a, b, c and d are real numbers is a *plane*.

Definition

The previous equality, true only for the points on the plane, is a *cartesian equation* of the plane.

Theorem

A plane has infinitely many different cartesian equations.

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1, 0, 0)$, $Q(\frac{1}{2}, 1, 0)$ and $R(0, 0, 1)$.

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1, 0, 0)$, $Q(\frac{1}{2}, 1, 0)$ and $R(0, 0, 1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\left\{ \begin{array}{rcl} a \times 1 + b \times 0 + c \times 0 & = & d \\ & & \\ & & \end{array} \right.$$

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1, 0, 0)$, $Q(\frac{1}{2}, 1, 0)$ and $R(0, 0, 1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ & & \end{cases}$$

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ a \times 0 + b \times 0 + c \times 1 & = & d \end{cases}$$

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ a \times 0 + b \times 0 + c \times 1 & = & d \end{cases}$$

From the first equation we get $a = d$,

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ a \times 0 + b \times 0 + c \times 1 & = & d \end{cases}$$

From the first equation we get $a = d$, and from the last $c = d$.

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ a \times 0 + b \times 0 + c \times 1 & = & d \end{cases}$$

From the first equation we get $a = d$, and from the last $c = d$. Then, the second equation is equivalent to

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ a \times 0 + b \times 0 + c \times 1 & = & d \end{cases}$$

From the first equation we get $a = d$, and from the last $c = d$. Then, the second equation is equivalent to $\frac{1}{2}d + b = d$, or

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ a \times 0 + b \times 0 + c \times 1 & = & d \end{cases}$$

From the first equation we get $a = d$, and from the last $c = d$. Then, the second equation is equivalent to $\frac{1}{2}d + b = d$, or $b = \frac{1}{2}d$.

How to find a cartesian equation

An example

Find a cartesian equation of the plane $\mathscr{P}$ passing through the points of coordinates $P(1,0,0)$, $Q(\frac{1}{2},1,0)$ and $R(0,0,1)$.

We know that a cartesian equation has the form

$$ax + by + cz = d.$$

As the points P, Q, R are on the plane, we deduce that a, b, c and d are solutions to the system

$$\begin{cases} a \times 1 + b \times 0 + c \times 0 & = & d \\ a \times \frac{1}{2} + b \times 1 + c \times 0 & = & d \\ a \times 0 + b \times 0 + c \times 1 & = & d \end{cases}$$

From the first equation we get $a = d$, and from the last $c = d$. Then, the second equation is equivalent to $\frac{1}{2}d + b = d$, or $b = \frac{1}{2}d$.

How to find a cartesian equation

As there are infininitely many cartesian equation, we can choose for d any number (except 0 in this case). Let's choose

How to find a cartesian equation

As there are infininitely many cartesian equation, we can choose for d any number (except 0 in this case). Let's choose $d = 2$.

How to find a cartesian equation

As there are infininitely many cartesian equation, we can choose for d any number (except 0 in this case). Let's choose $d = 2$.

Then we deduce $a = 2$, $c = 2$ and $b = 1$, so a cartesian equation of the plane is

How to find a cartesian equation

As there are infininitely many cartesian equation, we can choose for d any number (except 0 in this case). Let's choose $d = 2$.

Then we deduce $a = 2$, $c = 2$ and $b = 1$, so a cartesian equation of the plane is

$$2x + y + 2z = 2.$$

Parametric equations and lines

Definition

The set of points $M(x, y, z)$ in the 3D space such that

$$\begin{cases} x &= x_A + at \\ y &= y_A + bt \\ z &= z_A + ct \end{cases}$$

where x_A, y_A, z_A, a, b and c are real numbers, is a *line* passing through point A and with directing vector $\vec{u}(a, b, c)$.

Parametric equations and lines

Definition

The set of points $M(x, y, z)$ in the 3D space such that

$$\begin{cases} x = x_A + at \\ y = y_A + bt \\ z = z_A + ct \end{cases}$$

where x_A, y_A, z_A, a, b and c are real numbers, is a *line* passing through point A and with directing vector $\vec{u}(a, b, c)$.

Definition

The previous system, true only for the points on the line, is a set of *parametric equations* of the line.

Parametric equations and lines

Definition

The set of points $M(x, y, z)$ in the 3D space such that

$$\begin{cases} x &= x_A + at \\ y &= y_A + bt \\ z &= z_A + ct \end{cases}$$

where x_A, y_A, z_A, a, b and c are real numbers, is a *line* passing through point A and with directing vector $\vec{u}(a, b, c)$.

Definition

The previous system, true only for the points on the line, is a set of *parametric equations* of the line.

Theorem

A line has infinitely many different sets of parametric equations.

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1, 1, 0)$ and with directing vector $\vec{u}(0, 0, 1)$.

How to find parametric equations

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1, 1, 0)$ and with directing vector $\vec{u}(0, 0, 1)$.

According to the definition, a set of parametric equations of this line is

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1, 1, 0)$ and with directing vector $\vec{u}(0, 0, 1)$.

According to the definition, a set of parametric equations of this line is

$$\begin{cases} x = 1 + 0 \times t \\ \end{cases}$$

How to find parametric equations

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1, 1, 0)$ and with directing vector $\vec{u}(0, 0, 1)$.

According to the definition, a set of parametric equations of this line is

$$\left\{ \begin{array}{lcl} x & = & 1 + 0 \times t \\ y & = & 1 + 0 \times t \\ & & \end{array} \right.$$

How to find parametric equations

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1,1,0)$ and with directing vector $\vec{u}(0,0,1)$.

According to the definition, a set of parametric equations of this line is

$$\begin{cases} x &= 1 + 0 \times t \\ y &= 1 + 0 \times t \\ z &= 0 + 1 \times t \end{cases}$$

How to find parametric equations

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1,1,0)$ and with directing vector $\vec{u}(0,0,1)$.

According to the definition, a set of parametric equations of this line is

$$\left\{\begin{array}{lcl} x & = & 1+0\times t \\ y & = & 1+0\times t \\ z & = & 0+1\times t \end{array}\right. \text{ or } \left\{\begin{array}{lcl} x & = & 1 \\ & & \\ & & \end{array}\right.$$

How to find parametric equations

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1,1,0)$ and with directing vector $\vec{u}(0,0,1)$.

According to the definition, a set of parametric equations of this line is

$$\left\{\begin{array}{lcl} x & = & 1+0\times t \\ y & = & 1+0\times t \\ z & = & 0+1\times t \end{array}\right. \text{ or } \left\{\begin{array}{lcl} x & = & 1 \\ y & = & 1 \end{array}\right.$$

How to find parametric equations

An example

Find a set of parametric equations of the line $\mathscr{L}$ passing through the point of coordinates $T(1,1,0)$ and with directing vector $\vec{u}(0,0,1)$.

According to the definition, a set of parametric equations of this line is

$$\left\{\begin{array}{lll} x & = & 1+0\times t \\ y & = & 1+0\times t \\ z & = & 0+1\times t \end{array}\right. \text{ or } \left\{\begin{array}{lll} x & = & 1 \\ y & = & 1 \\ z & = & t \end{array}\right.$$

Intersections of lines and planes

Method

To find the coordinates of the intersection of a plane with a line (if it exists), just solve the system made of a cartesian equation of the plane and a set of parametric equations of the line.

Method

To find the coordinates of the intersection of a plane with a line (if it exists), just solve the system made of a cartesian equation of the plane and a set of parametric equations of the line.

- If the line intersects the plane in one just point, the solving will give one value for t, that can be used in the parametric equations to find the coordinates of the point.

Intersections of lines and planes

Method

To find the coordinates of the intersection of a plane with a line (if it exists), just solve the system made of a cartesian equation of the plane and a set of parametric equations of the line.

- If the line intersects the plane in one just point, the solving will give one value for t, that can be used in the parametric equations to find the coordinates of the point.
- If the line is parallel to the plane, then it will be impossible to solve the system.

Method

To find the coordinates of the intersection of a plane with a line (if it exists), just solve the system made of a cartesian equation of the plane and a set of parametric equations of the line.

- If the line intersects the plane in one just point, the solving will give one value for t, that can be used in the parametric equations to find the coordinates of the point.
- If the line is parallel to the plane, then it will be impossible to solve the system.
- If the line is included in the plane, then there will be an infinite number of solutions.

Intersections of lines and planes

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

To do so, we have to solve the system :

$$\begin{cases} 2x + y + 2z = 2 \\ \\ \end{cases}$$

Intersections of lines and planes

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

To do so, we have to solve the system :

$$\left\{ \begin{array}{rcl} 2x + y + 2z & = & 2 \\ x & = & 1 \\ y & = & 1 \\ z & = & t \end{array} \right.$$

We can replace x, y and z by their expressions as a function of t in the first equation. That gives :

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

To do so, we have to solve the system :

$$\begin{cases} 2x + y + 2z & = & 2 \\ x & = & 1 \\ y & = & 1 \\ z & = & t \end{cases}$$

We can replace x, y and z by their expressions as a function of t in the first equation. That gives :

$$2 \times 1 + 1 + 2t \quad = \quad 2$$

Intersections of lines and planes

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

To do so, we have to solve the system :

$$\begin{cases} 2x + y + 2z & = & 2 \\ x & = & 1 \\ y & = & 1 \\ z & = & t \end{cases}$$

We can replace x, y and z by their expressions as a function of t in the first equation. That gives :

$$\begin{aligned} 2 \times 1 + 1 + 2t &= 2 \\ 3 + 2t &= 2 \end{aligned}$$

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

To do so, we have to solve the system :

$$\begin{cases} 2x + y + 2z & = & 2 \\ x & = & 1 \\ y & = & 1 \\ z & = & t \end{cases}$$

We can replace x, y and z by their expressions as a function of t in the first equation. That gives :

$$\begin{aligned} 2 \times 1 + 1 + 2t &= 2 \\ 3 + 2t &= 2 \\ 2t &= -1 \end{aligned}$$

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

To do so, we have to solve the system :

$$\begin{cases} 2x + y + 2z & = & 2 \\ x & = & 1 \\ y & = & 1 \\ z & = & t \end{cases}$$

We can replace x, y and z by their expressions as a function of t in the first equation. That gives :

$$\begin{aligned} 2 \times 1 + 1 + 2t &= 2 \\ 3 + 2t &= 2 \\ 2t &= -1 \\ t &= -\frac{1}{2} \end{aligned}$$

An example

Find the coordinates of the intersection of the plane $\mathscr{P}$ with the line $\mathscr{L}$, if it exists.

To do so, we have to solve the system :

$$\begin{cases} 2x + y + 2z & = & 2 \\ x & = & 1 \\ y & = & 1 \\ z & = & t \end{cases}$$

We can replace x, y and z by their expressions as a function of t in the first equation. That gives :

$$\begin{aligned} 2 \times 1 + 1 + 2t &= 2 \\ 3 + 2t &= 2 \\ 2t &= -1 \\ t &= -\frac{1}{2} \end{aligned}$$

Intersections of lines and planes

Then, we use this value of t and the parametric equations to find the coordinates of the point :

$$\left\{ \begin{array}{lcl} x & = & 1 \\ & & \end{array} \right.$$

Intersections of lines and planes

Then, we use this value of t and the parametric equations to find the coordinates of the point :

$$\begin{cases} x &= 1 \\ y &= 1 \\ & \end{cases}$$

Intersections of lines and planes

Then, we use this value of t and the parametric equations to find the coordinates of the point :

$$\begin{cases} x &= 1 \\ y &= 1 \\ z &= -\frac{1}{2} \end{cases}$$

So the plane and the line intersection in the point of coordinates

$$\left(1, 1, -\frac{1}{2}\right).$$