

<b>Introduction to logic</b>	Season	01
	Episode	AP07
	Time frame	1 period

**Prerequisites :** None.

**Objectives :**

- Discover the concepts of implication, converse and contrapositive.

**Materials :**

- *Implications on cards.*
- *Answer sheet.*
- *Exercise sheet.*
- *Slideshow.*

### **1 – Matching game**

10 mins

Each student in the class is given a card with an implication on it. Students mingle to find :

1. first, the converse of their implication ;
2. second, the contrapositive of their implication.

Using a slideshow, the teacher introduces the vocabulary of logic.

### **2 – True or not**

10 mins

Students gather in groups of 4 : an implication, its converse, its contrapositive and the converse of the contrapositive. Together, they discuss which ones of the four sentences are true.

### **3 – Exercises**

Remaining time

Still working in groups, students have to solve a few exercises about implication.

Surname		First name		Form	
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	Document	Answer sheet

## Your four implications

Type	Sentence	True ?
Implication		
Contrapositive		
Converse		
Contrapositive of the converse		

## Exercises

### Exercise 1

For each of the following implications, write its converse then decide if the initial implication is true and if the converse is true.

Type	Sentence	True ?
Implication	If a triangle $ABC$ is inscribed in a circle of diameter $[AB]$ , then it's right-angled in $C$ .	
Converse		
Implication	If a line passes through the midpoints of two sides of a triangle, then it's parallel to the third side.	
Converse		

Type	Sentence	True?
Implication	If a triangle has an altitude that is also a median, then it's isosceles.	
Converse		
Implication	If two altitudes of a triangle meet in a point $H$ , then the third altitude also passes through $H$ .	
Converse		
Implication	If a product is equal to 0, then at least one of the terms is equal to 0.	
Converse		

### Exercise 2

For each pair of statements  $p$  and  $q$ , tick the propositions that are true.

$p$  : I live in France.

$p \Rightarrow q$         $q \Rightarrow p$

$q$  : I live in Europe.

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  : I am overage.

$p \Rightarrow q$         $q \Rightarrow p$

$q$  : I'm 19.

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $CDEF$  is a parallelogram.

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $CDEF$  is a square.

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $x \in \mathbf{N}$ .

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $x \in \mathbf{Z}$ .

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $MNP$  is right-angled in  $M$ .

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $MP^2 + MN^2 = NP^2$ .

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $x \geq -2$ .

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $x \geq -1$ .

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $a + b = 5$ .

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $a = 2$  and  $b = 3$ .

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $4x - (x + 5) = 7$ .

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $x = 4$ .

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $n$  is prime.

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $n$  is not a multiple of 3.

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

$p$  :  $(ax + b)(cx + d) = 0$ .

$p \Rightarrow q$         $q \Rightarrow p$

$q$  :  $ax + b = 0$  or  $cx + d = 0$ .

$p \Leftrightarrow q$         $\neg p \Rightarrow \neg q$         $\neg q \Rightarrow \neg p$

**Document 1** Implications

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If the triangle  $ABC$  is right-angled in  $A$ , then  $BC^2 = AB^2 + AC^2$ .

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If  $BC^2 = AB^2 + AC^2$ , then the triangle  $ABC$  is right-angled in  $A$ .

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If the triangle  $ABC$  is not right-angled in  $A$ , then  $BC^2 \neq AB^2 + AC^2$ .

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If  $BC^2 \neq AB^2 + AC^2$ , then the triangle  $ABC$  is not right-angled in  $A$ .

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If a quadrilateral is a square,  
then its diagonals have the same length.

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If the diagonals of a quadrilateral have the same length,  
then it's a square.

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If a quadrilateral is not a square,  
then its diagonals don't have the same length.

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If the diagonals of a quadrilateral don't have the same length,  
then it's not a square.

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If  $x$  is a real number such that  $x^2 = 9$ , then  $x = 3$ .

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If  $x$  is a real number such that  $x = 3$ , then  $x^2 = 9$ .

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If  $x$  is a real number such that  $x^2 \neq 9$ , then  $x \neq 3$ .

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If  $x$  is a real number such that  $x \neq 3$ , then  $x^2 \neq 9$ .

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If the median of a statistical set of data is 12,  
then at least 50% of the values are greater than or equal to 12.

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If at least 50% of the values are greater than or equal to 12,  
then the median of a statistical set of data is 12.

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If the median of a statistical set of data is not equal to 12,  
then less than 50% of the values are greater than or equal to 12.

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If less than 50% of the values are greater than or equal to 12,  
then the median of a statistical set of data is not equal to 12.

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If a triangle is equilateral, then it's right-angled.

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If a triangle is right-angled, then it's equilateral.

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If a triangle is not equilateral, then it's not right-angled.

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If a triangle is not right-angled, then it's not equilateral.

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