Congruent triangles	Season	01
	Episode	AP09
	Time frame	1 period

Prerequisites : Vocabulary and notions about triangles and angles.

Objectives :
- Discover the concept of congruent triangles.
- Apply out the three congruence cases to prove that some triangles are congruent.

Materials :
- *Slideshow.*
- *Lesson.*
- *Exercises.*
- *Slideshow with the correction of the exercises.*

1 – Lesson   15 mins

Using a slideshow, the teacher explains the concept of congruent triangles and the three congruence cases. Students should have an active part to the lesson. At the end, a lesson sheet is handed out.

2 – Exercises   Remaining time

Working in groups, students have to work on a few exercises about congruent triangles.
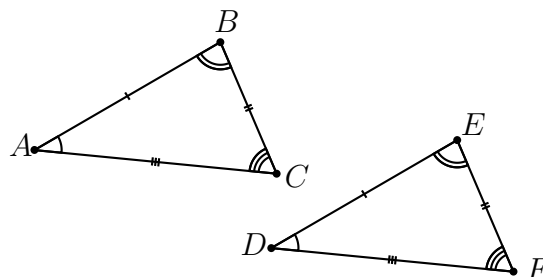
Definition 1 Equal sides

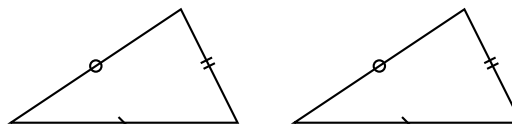Two triangles are congruent if they have three pairs of equal sides.

Proposition 1 Equal angles

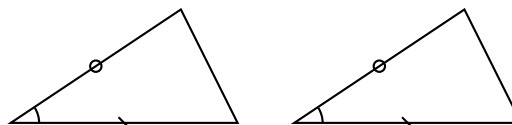Two congruent triangles have three pairs of equal angles.

To prove that two triangles are congruent, one can use one of the following methods :

SSS : Prove that the triangles have three pairs of equal sides.

SAS : Prove that the triangles have two pairs of equal sides and the angles between these sides equal.

ASA : Prove that the triangles have two pairs of equal angles and the sides between these sides equal.

Exercise 1

Let $ABCD$ be a parallelogram.

1. Prove that triangles ABC and CDA are congruent.

2. Find another pair of congruent triangles on this figure.

3. Let O be the intersection point of the diagonals of $ABCD$. Find two pairs of congruent triangles involving the point O. Prove these properties.

Exercise 2

Let ABC be an isosceles triangle with main vertex B, and H the foot of the altitude through B. Find two congruent triangles in the figure and prove the property.

Exercise 3

Let $ABCD$ be an isosceles trapezium, where
- the sides AD and BC are parallel;
- the sides AB and CD are equal;
- the angles ABC and BCD are equal.
Let I be the midpoint of the side BC.

1. Prove that the triangles ABI and CDI are congruent.

2. Deduce a property about triangle ADI.

Exercise 4

Let ABC be an equilateral triangle, with M, N, P three points respectively on the sides BC, CA and AB such that $BM = CN = AP$.
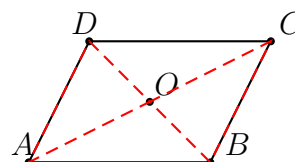
1. Prove that the triangles BMP, CNM and APN are congruent to one another.

2. Deduce a property about triangle MNP.
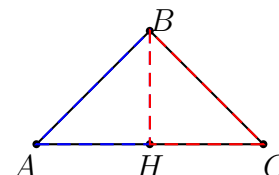
Exercise 1

1. • $AB = DC$ because $ABCD$ is a parallelogram.
   • $BC = DA$ because $ABCD$ is a parallelogram.
   • $AC = AC$ because it's a commun side.
   so the triangles ABC and CDA have three pairs of equal sides, so they are congruent.

2. The triangles ABD and BCD are congruent too.

3. • $AO = OC$ because O is the midpoint of the diagonals.
   • $DO = OB$ because O is the midpoint of the diagonals.
   • $AD = BC$ because $ABCD$ is a parallelogram.
   so the triangles AOD and COB have three pairs of equal sides, so they are congruent.
   • $AO = OC$ because O is the midpoint of the diagonals.
   • $DO = OB$ because O is the midpoint of the diagonals.
   • $AB = DC$ because $ABCD$ is a parallelogram.
   so the triangles AOB and COD have three pairs of equal sides, so they are congruent.
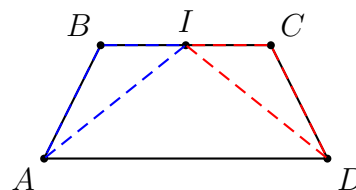
Exercise 2

• $BH = BH$ because it's a common side.
• $AB = CB$ because $\triangle ABC$ is isosceles.
• $\angle BAH = \angle BCH$ because $\triangle ABC$ is isosceles.

So the triangles ABC and CDA have two pairs of equal sides and the angles between these sides equal, so they are congruent.

Exercise 3

1. • $CI = BI$ because I is the midpoint of BC.
   • $AB = CD$ because $ABCD$ is an isosceles trapezium.
   • $\angle ABI = \angle ICD$ because $\triangle ABC$ is isosceles.
   So $\triangle ABC$ and $\triangle CDA$ have two pairs of equal sides and the angles between these sides equal, so they are congruent.
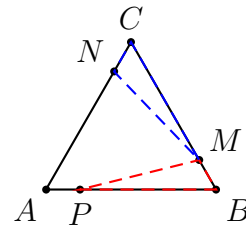
2. Since the triangles ABC and CDA are congruent, the third sides are also equal so $AI = ID$. Consequently $\triangle ADI$ is isosceles.

Exercise 4

1. • $BM = CN$ according to the instructions.
   • $BP = CM$ because $BP = side - AP = side - BM = CM$.
   • $\angle PBM = \angle MCN$ because the triangle ABC is equilateral.

   So $\triangle BMP$ and $\triangle CNM$ have two pairs of equal sides and the angles between these sides equal, so they are congruent.

2. In the same way, we prove that $\triangle CNM$ and $\triangle APN$ are congruent.

3. We can deduce that the third sides of the triangles are equal, so $\triangle MNP$ is equilateral.