# Homework #2

In this exercise, we study *Koch's snowflake*, a figure with surprising properties.

## Part A – Construction of the figure

This construction can be done by hand or using a software such as Geogebra. Draw each new figure as a new one, and not on the previous one.
To create a Koch snowflake, start with an equilateral triangle of side 9cm, then recursively alter each line segment as follows :

1. Divide the line segment into three segments of equal length.
2. Draw an equilateral triangle that has the middle segment from step 1 as its base and points outward.
3. Remove the line segment that is the base of the triangle from step 2.

This process can theoretically go on ad infinitum, but you will stop after two replacement steps. In this way, you should end up with 3 different figures, including the initial triangle.

## Part B – Study of the perimeter

We call $\mathscr{P}(n)$ the perimeter of the figure after $n$ steps. For example, $\mathscr{P}(0)$ is the perimeter of the initial triangle, and $\mathscr{P}(2)$ is the perimeter of the figure after 2 steps, the last one you drew.

1. Compute $\mathscr{P}(0)$, the perimeter of the initial triangle. Explain your computation.
2. Compute $\mathscr{P}(1)$ justifying every part of your computation.
3. Compute $\mathscr{P}(2)$ justifying every part of your computation.
4. Compute the ratios $\frac{\mathscr{P}(1)}{\mathscr{P}(0)}$ and $\frac{\mathscr{P}(2)}{\mathscr{P}(1)}$. What do you notice ?
5. Prove that each replacement step multiplies the perimeter of the figure by $\frac{4}{3}$.
6. Deduce the perimeters of $\mathscr{P}(3)$, $\mathscr{P}(4)$, $\mathscr{P}(5)$ and $\mathscr{P}(6)$. Give the answers as exact values and rounded values to 3 DP.
7. Use your calculator to find a value of $n$ such that $\mathscr{P}(n) > 250$.
8. What can you say about $\mathscr{P}(n)$ when $n$ gets greater and greater ? Do you think there is a maximum value for $\mathscr{P}(n)$ ?

## Part C – Study of the area

We call $\mathscr{A}(n)$ the area of the figure after $n$ steps. For example, $\mathscr{A}(0)$ is the area of the initial triangle, and $\mathscr{A}(2)$ is the area of the figure after 2 steps, the last one you drew.

1. Find the formula for the area of an equilateral triangle of side $s$ cm. Explain your method.
2. Use the formula to compute $\mathscr{A}(0)$, the area of the initial triangle. Give the exact value and a rounded value to 3 DP.
3. Use the formula to compute the area of an equilateral triangle of side 3 cm and deduce the value of $\mathscr{A}(1)$. Give the exact value and a rounded value to 3 DP.

4. Use the formula to compute the area of an equilateral triangle of side 1 cm and deduce the value of $\mathscr{A}(2)$. Give the exact values and rounded values to 3 DP.
5. Compute the differences $\mathscr{A}(1) - \mathscr{A}(0)$ and $\mathscr{A}(2) - \mathscr{A}(1)$. Give the exact value and a rounded value to 3 DP. What do you notice ?
6. Do you think the area of the figure will grow like its perimeter ?
7. Find on the internet a proof of the fact that the area of the figure stays finite.
8. What is the most surprising feature of about Koch's snowflake ?