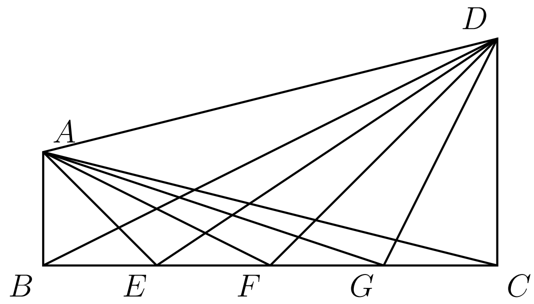
Homework #7

In the figure below, the angles $\widehat{ABC}$ and $\widehat{BCD}$ are right and, in units, $AB = BE = EF = FG = GC = 1$, $BC = 4$ and $CD = 2$. The segments linking the vertices A and D to every point on the segment $[BC]$ have been drawn.



Part A – Static experiments with Geogebra

1. Reproduce the picture using Geogebra.
2. (a) Use Geogebra to compute the lengths AE, AF, AG and AC. Give in each case an approximation to 3DP.
   (b) Do the same for the lengths DB, DE, DF and DG.
   (c) Two of the eight lengths you've computed are equal. Explain why.
3. (a) Use Geogebra to compute the sums $AB+BD$, $AE+ED$, $AF+FD$, $AG+GD$ and $AC+CD$ to 3DP.
   (b) Which of these 5 sums is the lowest ? Do you find this surprising ?

Part B – Dynamic experiments with Geogebra

1. Do a new figure with Geogebra, with only the points A, B, C, D and the segments $[AB]$, $[BC]$ and $[CD]$
2. Define the lengths $m = AM$ and $n = MD$ and then their sum $d = m + n$.
3. Ask Geogebra to show the values of the numbers to 5 DP, and move M along $[BC]$ to find the position where d is minimal. Zoom in if you need more precision.
4. Write down the value of BM such that d is minimal. This value is close to a simple fraction, which one ?

Part C – Using functions

In this part, we name x the distance BM and $d(x)$ the sum of distances $AM + MD$.

1. Write the lengths MC, AM, and MD as functions of x.
2. Give the expression of $d(x)$ as a function of x.
3. Draw the graph of this function with Geogebra or with your calculator. Print it or, if you can't, draw it on your paper.

4. Find graphically, and with 5 DP, the minimum of the function d and the values of x for which this minimum is reached.
5. Explain how the result of the previous question confirms the conclusion of part A.

Part D – Some properties of the minimal point

Let E be the point on the segment $[BC]$ such that $\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BC}$.

1. Do a new figure with Geogebra and place precisely the point E. Print it or copy it on your paper at the end of the exercise.
2. What can you say about the directions of the vectors $\overrightarrow{BE}$ and $\overrightarrow{CE}$? Compute their norms (lengths) as fractions.
3. What can you deduce about the sum $2\overrightarrow{BE} + \overrightarrow{CE}$?
4. Use the definition of E and Chasles' relation to prove again the equality you found in the previous question.
5. Use Geogebra to build the point A', symmetric of A around the point B.
6. Draw the line $(A'D)$. What do you notice ?
7. What is the position of a point M on the segment $[BC]$ such that the sum $A'M+MD$ is minimal ? Deduce an explanation of the property you noticed in the previous question.