Homework

Galileo Galilei (15 February 1564 - 8 January 1642) was an Italian physicist, mathematician, astronomer and philosopher, and withouth any doubt one of the greatest minds of all times. He was born, the same year as Shakespeare, in the Italian town of Pisa. He was sent to the University of Pisa to study medicine, but in 1589 became professor of mathematics (at the age of 25) through the favours of Ferdinando dei Medici, the Grand Duke of Tuscany. During this period, Galileo was "ordered" by the Grand Duke of Tuscany to explain a paradox arising in the experiment of tossing three dice.

> *Why, although there were an equal number of 6 partitions of the numbers 9 and 10, did experience state that the chance of throwing a total 9 with three fair dice was less than that of throwing a total of 10 ?*

Part A – Simulating 1000 throws

Use a spreadsheet (OpenOffice Calc or Microsoft Excel, for example) to simulate 1000 throws of a fair die, and count the frequency of each result. To do so, you may use the following functions :

`=ALEA.ENTRE.BORNES(1;N)` **:** Produces a random integer between 1 and N.

`=SOMME(A1:A7)` **:** Computes the sum of the numbers in cells A1 to A7, including all cells in between.

`=NB.SI(A1:A7;k)` **:** Counts how many times the value k occurs in the cells A1 to A7.

Give the results of your 1000 simulations in an absolute frequency table. Does it confirm the Duke's statement ?

Part B – Galileo's solution

Below is an extract from Galileo's answer to the Great Duke of Tuscany, translated by E. H. Thorne.

> *The fact that in a dice-game certain numbers are more advantageous than others has a very obvious reason, i.e. that some are more easily and more frequently made than others, which depends on their being able to be made up with more variety of numbers. Thus a 3 and an 18, which are throws which can only be made in one way with 3 numbers (that is, the latter with 6.6.6 and the former with 1.1.1, and in no other way), are more difficult to make than, e.g. 6 or 7, which can be made up in several ways, that is, a 6 with 1.2.3 and with 2.2.2 and with 1.1.4, and a 7 with 1.1.5, 1.2.4, 1.3.3, and 2.2.3. Nevertheless, although 9 and 12 can be made up in as many ways as 10 and 11, and therefore they should be considered as being of equal utility to these, yet it is known that long observation has made dice-players consider 10 and 11 to be more advantageous than 9 and 12. And it is clear that 9 and 10 can be made up by an equal diversity of numbers (and this is also true of 12 and 11). [...]*
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> *Three special points must be noted for a clear understanding of what follows. The first is that that sum of the points of 3 dice, which is composed of 3 equal numbers, can only be produced by one single throw of the dice : and thus a 3 can only be produced by the*

three ace-faces, and a 6, if it is to be made up of 3 twos, can only be made by a single throw. Secondly : the sum which is made up of 3 numbers, of which two are the same and the third different, can be produced by three throws : as e.g., a 4 which is made up of a 2 and of two aces, can be produced by three different throws; that is, when the first die shows 2 and the second and third show the ace, or the second die a 2 and the first and third the ace; or the third a 2 and the first and second the ace. And so e.g., an 8, when it is made up of 3.3.2, can be produced also in three ways : i.e. when the first die shows 2 and the others 3 each, or when the second die shows 2 and the first and third 3, or finally when the third shows 2 and the first and second 3. Thirdly the sum of points which is made up of three different numbers can be produced in six ways. As for example, an 8 which is made up of 1.3.4. can be made with six different throws : first, when the first die shows 1, the second 3 and the third 4; second, when the first die still shows 1, but the second 4 and the third 3; third, when the second die shows 1, and the first 3 and the third 4; fourth, when the second still shows 1, and the first 4 and the third 3; fifth, when the third die shows 1, the first 3, and the second 4; sixth, when the third shows 1, the first 4 and the second 3. Therefore, we have so far declared these three fundamental points; first, that the triples, that is the sum of three-dice throws, which are made up of three equal numbers, can only be produced in one way; second, that the triples which are made up of two equal numbers and the third different, are produced in three ways; third, that those triples which are made up of three different numbers are produced in six ways.

1. In the first paragraph, Galileo explains that there is only one way to make a 3, but several to make a 6.
   (a) Find in the text how many ways there are to make a 6.
   (b) In the same manner, find out all the ways to make the numbers 9 and 10.
   (c) What statement in Galileo's text is then proved?
2. Explain the second paragraph with the example of the different ways to make a 6, and deduce how many different throws actually make this value.
3. Compute in the same way the number of different throws that make a 9 and those that make a 10.
4. Write the conclusion of Galileo's paper, explaining to the Great Duke of Tuscany the solution to this problem. (Minimum 50 words, maximum 150 words.)

Part C – The Passe-Dix game

Passe-dix, also called passage in English, is a game of chance using dice. It is played with three dice. There is always a banker, and the number of players is unlimited. To win, a player must throw a point above ten (or pass ten – whence the name of the game).

1. Use Galileo's method to compute the number of throws that can give each possible sum. Give the results in an absolute frequency table.
2. Add to the previous table the probabilities of each sum, given as an irreducible fraction and an approximate value to 3 DP.
3. Compute the relative frequencies in the 1000 simulated throws of part A and compare them to the probabilities. Are the values very different?
4. Compute the probability of "passing ten". Is it a fair game?