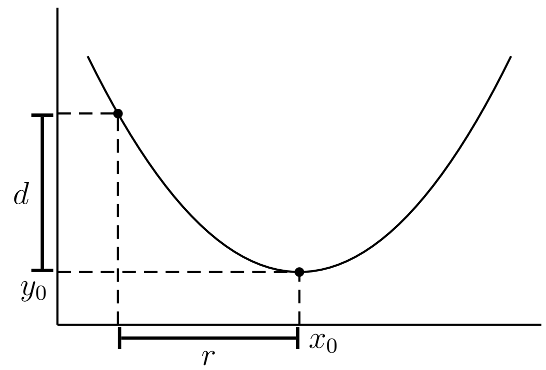
Homework # 11

An interesting formula
In this homework, we will look at a formula that allows to find the formula of a polynomial function of the second degree, knowing only the coordinates of its extremal point and two special characteristics of the curve.

In the picture on the right-hand side, (x_0, y_0) are the coordinates of the extremal point, and r and d are the horizontal and vertical distances between the two points. In the picture, x_0 and y_0 are positive, and they could also be negative.
The number r is called the *radius* and d the *deviation*.



Partie A – An example

In this part, we will try to find the formula of a quadratic function whose extremal point is $(3, 7)$ with a deviation of 2 for a radius of 4.

1. Find the simplest linear function such that the image of 3 is 0.
2. Turn the previous function into a quadratic function such that the image of 3 is 0.
3. Deduce a quadratic function g such that the image of 3 is 7.
4. (a) Use the radius and the deviation to find another point on the curve.
   (b) The ordinate of the new point should be the image of its abscissa under function g. Check that it is so with the formula found previously.
   (c) With only one multiplicative coefficient, turn the formula of function g into the formula of a new function f that works for the two points.
5. By using the radius and deviation symmetrically, we get the coordinates of another point that should be on the curve. Check that the formula works for this point too.
6. Expand the formula you found for f in the previous question.
7. Draw the variations table of the function f, and then its sign table.

Partie B – The general formula

In this part, we don't know the coordinates (x_0, y_0), the radius r nor the deviation d. We will try to find a general formula.

1. Find the simplest linear function such that the image of x_0 is 0.
2. Turn the previous function into a quadratic function such that the image of x_0 is 0.
3. Deduce a quadratic function g such that the image of x_0 is y_0.
4. (a) Use the radius and the deviation to find another point on the curve.
   (b) The ordinate of the new point should be the image of its abscissa under function g. Check that it is so with the formula found previously.

(c) Using the multiplicative coefficient $\frac{d}{r^2}$, turn the formula of function g into the formula of a new function f that works for the two points.

5. By using the radius and deviation symmetrically, we get the coordinates of another point that should be on the curve. Check that the formula works for this point too.

Partie C – Applying the formula

Use the formula to find the quadratic functions with the characteristics show below. In each case, give the initial expression, then the expanded one and the variations table.

1. $x_0 = -2$, $y_0 = 4$, $r = 4$ and $d = -2$.
2. $x_0 = 7$, $y_0 = -2$, $r = 5$ and $d = 3$.
3. $x_0 = 4$, $y_0 = 6$, $r = 3$ and $d = -5$.