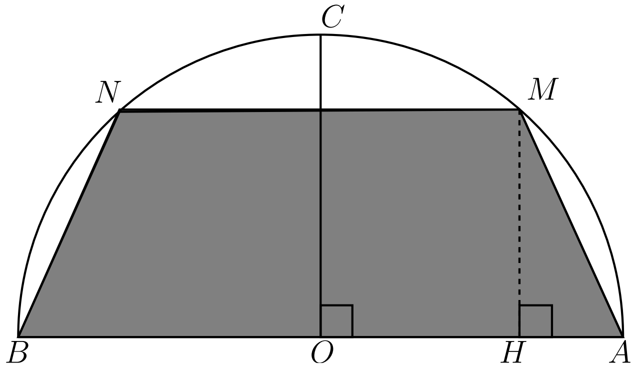
Homework #11

In the figure below, $[AB]$ is the diameter of the half-circle $\mathscr{C}$ and its length is worth 8cm.O is the midpoint of $[AB]$ and C is the midpoint of the arc $\widehat{AB}$.
Let M a moving point onto the arc $\widehat{AC}$. The parallel to (AB) through M cut $\mathscr{C}$ in N and H is the projection of M on (AB).
The aim of this homework is to find the position of the point M that maximize the perimeter of the trapezoid $AMNB$.



Partie A : Geometric simulation

1. Use Geogebra to draw the figure.
2. Move the point M onto the arc $\widehat{AC}$.
   Surmize the maximal value of the perimeter of $AMNB$.

Partie B : Using a function

1. Let x represent the length of AM.
   (a) Compute the value of AC. Deduce the intervall of the values of x.
   (b) Prove that $BN = x$.
2. Let $AH = a$.
   Use Pythagorean theorem in two wise-chosen triangles, prove successively that :
   (a) $MH^2 = 16 - (4-a)^2$;
   (b) $MH^2 = x^2 - a^2$.
3. Deduce from the previous questions that $a = \dfrac{x^2}{8}$ and then that $MN = 8 - \dfrac{x^2}{4}$.
4. Établir que le périmètre du trapèze $AMNB$ est donné en fonction de x par l'expression $-\dfrac{x^2}{4} + 2x + 16$.
5. Let f the function defined over $[0; 6]$ by : $f(x) = -\dfrac{x^2}{4} + 2x + 16$.
   (a) Graph, using Geogebra, the function f.
   (b) According to the graph of f, draw the table of variation of the function.
   (c) Surmize the position of M for which the perimeter of f is maximal.