| **Combinatorics** | Season | 01 |
| | Episode | 12 |
| | Time frame | 1 period |

**Prerequisites :** Events and probabilities

**Objectives :**
● Discover a few combinatorics methods

**Materials :**
● *Exercise sheet for each student.*
● *List of questions for the quizz.*

## 1 – Group work                                                    25 mins

Students work in groups of 4. They have to solve an exercise involving two standard combinatorics situations and the factorial notation.

## 2 – Quizz                                                         30 mins

Students are still working in groups. They have to answer ten simple combinatorics questions (3 minutes per question). At the end of the quizz, group papers are marked by the teacher.

There are 26 letters in the latin alphabet we use, from a to z. A word is an ordered sequence of letters, with possible repetitions. In the first part of this exercise we will count the number of possible words, without considering the fact that a word as a meaning or not. For example, "yyyyryyyr" is a 9-letters word, just like "pentagone".

**1.** How many possible 2-letters words are there ?

**2.** How many possible 3-letters words are there ?

**3.** How many possible 4-letters words are there ?

**4.** Find a general formula for the number of possible $n$-letters words, with $n$ a natural number.

**5.** The longest English word appearing in a dictionnary is

<p align="center">"Pneumonoultramicroscopicsilicovolcanoconiosis"</p>

It's a disease of the lungs, caused by inhaling mineral or metallic dust, such as silicon and quartzite.
Count the number of possible words with the same number of letters.

**6.** The total number of possible words with 46 letters of less is approximately equal to $1.275872703 \times 10^{65}$. Imagine that we want to create a dictionnary listing all these possible words, and that we manage to put 500 words in each page. Now, suppose we use a paper with a thickness of 0,01 millimeters (which is very thin).
Compute the number of pages needed and total thickness of the dictionnary, using the appropriate unit.

In the second part, we will study a different problem : the number of anagrams for a given word. An anagram is the result of rearranging the letters of a word or phrase to produce a new word or phrase, using all the original letters exactly once. An interesting example is "I'm a dot in place", an anagram of "A decimal point" ; a famous one is "I Am Lord Voldemort", an anagram of "Tom Marvolo Riddle". In the following questions, we will consider all anagrams, whether they mean something or not. For example, "fnctnuio" is an anagram of "function".

**1.** List all the anagrams of the words "no", "yes" and "blue".

**2.** Find out the number of anagrams of any 2-letters word, any 3-letters word and any 4-letters word, counting in each case the word itself.

**3.** The notation $n!$ denotes the product of all integers from 1 to $n$. Use this notation to write the answers of the previous question.

**4.** Prove that the number of anagrams of any 5-letters word is 5!.

**5.** What is the number of anagrams of any $n$-letters word.

## Document 1 Quizz

1. For every meal, a student has to choose between 6 starters, two main courses, two sides, 5 cheeses or yoghurt and 3 desserts. What is the number of possible meals ?

2. A man wears only black and white. Every morning he picks randomly one pair of shoes (black or white), one pair of trousers (black or white), one shirt (black or white) and one overcoat (black or white). What is the number of possible outfits ?

3. In horse racing terminology, a trifecta is a parimutuel bet in which the bettor must predict which horses will finish first, second, and third in exact order. If there are 12 horses in the race, how many different choices are there ?

4. Every morning, a man living in London picks randomly an umbrella before going out in the rain. He can choose between three different ones : one green, one red and one black. What is the number of possible choices for one week.

5. Mario Kart Wii is a famous racing game developed by Nintendo. In this game, the player must choose between 4 modes, 24 Nintendo characters, 8 vehicules for each character, 8 cups and 4 races per cup. What is the number of possible races ?

6. A group of 120 people have to elect a board made of a Chairman, a vice-Chairman and a secretary. How many possible boards are there ? Anyone can be elected, but a different person must be elected for each post.

7. Every week, a teacher picks randomly a pupil in a class of 36 to wipe away the board after each lesson. What is the number of possible choices for 5 weeks ?

8. Ten people have to be arrange in a line and handed out numbers from 1 to 10. What is the number of possible arrangements ?

9. How many distinct car licence plates are there consisting of seven characters, the first two and last two being letters and the middle ones digits ?

10. As I was going to St Ives
   I met a man with seven wives
   Each wife had seven sacks
   Each sack had seven cats
   Each cat had seven kits
   Kits, cats, sacks, wives
   How many were going to St Ives ?