

Épreuve de section européenne

1 General knowledge

Which types of triangles do you know? give a few properties.

2 Document

We consider problems concerning the number of ways in which a number can be written as a sum. If the order of the terms in the sum is taken into account the sum is called a *composition* and the number of compositions of n is denoted by $c(n)$. If the order is not taken into account the sum is a *partition* and the number of partitions of n is denoted by $p(n)$.

Thus, the compositions of 3 are

$$3 = 3; 3 = 1 + 2; 3 = 2 + 1 \text{ and } 3 = 1 + 1 + 1;$$

so that $c(3) = 4$.

The partitions of 3 are

$$3 = 3; 3 = 2 + 1 \text{ and } 3 = 1 + 1 + 1;$$

so $p(3) = 3$.

[...]

The function $c(n)$ is easily determined as follows. Consider n written as a sum of 1's. We have $n - 1$ spaces between them and in each of the spaces we can insert a slash, yielding 2^{n-1} possibilities corresponding to the 2^{n-1} compositions of n . For example

$$3 = 111; 3 = 1/11; 3 = 11/1; 3 = 1/1/1.$$

From *An Introduction to the theory of numbers*, by Leo Moser.

3 Questions

1. According to this text, what is the number of compositions of a natural number n ?
2. Compute the number of compositions of 2, 3, 4 and 5. Do these numbers agree with the formula given in the text?
3. Explain the method used in the text to prove the formula. In particular, explain the role of the slash used at the end of the text.