

Épreuve de section européenne

1 General knowledge

Using an integration by parts, calculate $\int_1^x \ln t dt$, where $x > 0$.

2 Document

The Theory of Complex Numbers may be said to have attracted attention as early as the sixteenth century in the recognition, by the Italian algebraists, of imaginary or impossible roots. In the seventeenth century Descartes distinguished between real and imaginary roots, and the eighteenth saw the labors of De Moivre and Euler. To De Moivre is due (1730) the well-known formula which bears his name, $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$, and to Euler (1748) the formula $\cos \theta + i \sin \theta = e^{\theta i}$.

The geometric notion of complex quantity now arose, and as a result the theory of complex numbers received a notable expansion. The idea of the graphic representation of complex numbers had appeared, however, as early as 1685, in Wallis's *De Algebra tractatus*. [...] In 1804 the Abbé Buée independently came upon the same idea which Wallis had suggested, that $\pm\sqrt{-1}$ should represent a unit line, and its negative, perpendicular to the real axis. Buée's paper was not published until 1806, in which year Argand also issued a pamphlet on the same subject. It is to Argand's essay that the scientific foundation for the graphic representation of complex numbers is now generally referred.

From *History of Modern Mathematics*, by David Eugene Smith.

3 Questions

1. When do the first works about complex numbers date back to?
2. How did Descartes use complex numbers?
3. Use the various notations of a complex number to prove De Moivre's formula.
4. Name three mathematicians who independently devised a way to graphically represent complex numbers.
5. What do we sometimes call an Argand's diagram? Draw one and place the images of some important complex numbers.