

Mathematics in english

1 Document

From “Algebra” by M. Artin, Appendix, Section 2

Exactly what mathematicians consider an appropriate way to present a proof is not clearly defined. It isn't customary to give proofs which are complete in the sense that every step consists in applying a rule of logic to the previous step. Writing such a proof would take too long, and the main points wouldn't be emphasized. On the other hand, all difficult steps of the proof are supposed to be included. Someone reading the proof should be able to fill in as many details as needed to understand it. How to write a proof is a skill that can be learned only by experience.

We will discuss three important techniques used to construct proofs : *dichotomy, induction and contradiction*.

The word *dichotomy* means division into parts. It is used to subdivide a problem into smaller, more easily managed pieces.[...]

Induction is the main method for proving a sequence of statements P_n , indexed by positive integers. To prove P_n for all n , the principle of induction requires us to do two things : (i) prove that P_1 is true, and (ii) prove that if, for some integer $k > 1$, P_k is true, then P_{k+1} is also true.[...]

Proofs by contradiction proceed by assuming that the desired conclusion is false and deriving a contradiction from this assumption.[...]

2 Questions

1. According to the author, is it possible or desirable to write a complete proof for every mathematical statement ?
2. What parts of a proof can be omitted ?
3. How can the skill of writing of a proof be learned, according to M. Artin ?
4. What are the three main techniques of proof ?
5. Explain shortly each technique.