Épreuve de section européenne

1 General knowledge

How can you round numbers ? Give a few definitions and a few examples.

2 Document

Kobon Fujimura, a Japanese puzzle expert, recently invented a problem in combinatorial geometry. It is simple to state, but no general solution has yet been found. What is the largest number of non-overlapping triangles that can be produced by n straight line segments ? It is not hard to discover by trial and error that for $n = 3$, 4, 5 and 6 the maximum number of triangles is respectively one, two, five and seven. For seven lines the problem is no longer easy.

Saburo Tamura made progress on the Kobon Triangle problem by proving that the whole part of $\frac{n(n-2)}{3}$ was an upper bound on the maximal number of non-overlapping triangles realizable by n lines.

The various solutions known at the beginning of 2006 are given in the following table, where the second line gives the actual maximum number of triangles obtained so far, and the third line gives the theoretical upper bound proved by Saburo Tamura.

n	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Best known	1	2	5	7	11	15	21	25	32	28	47	?	65	?
Upper bound	1	2	5	8	11	16	21	26	33	40	47	56	65	74

Adapted from a *www.maa.org* article by Ed Pegg Jr.

3 Questions

1. Draw a solution for $n = 3$, 4 and 5.
2. What is an upper bound ? Explain the notion in the context this article.
3. Let (u_n) be the sequence defined by $u_n = \frac{n(n-2)}{3}$. Prove that the sequence (w_n) defined by $w_n = u_{n+1} - u_n$ is an arithmetic progression and give the difference between two successive numbers. What does it mean for the number of Kobon triangles ?
4. For what values of n has the problem been solved for sure ?