Épreuve de section européenne

1 General knowledge

Give the definition of a circle. Give some vocabulary about circles and state the properties you know.

2 Document

Let A and B be any sets. Then a mapping θ of A into B is determined when there is a rule or formula which assigns, to each element x of A, an element of B, called the image of x under θ. This element of B is usually written $\theta(x)$. It is convenient to use the notation $\theta : A \longrightarrow B$ to indicate that θ is a mapping of A into B. A is called the domain of θ and B is called the range.

Example : We can define a mapping θ from the set P of all people into the set of all integers, by the rule : if $x \in P$, then $\theta(x)$ is the age of x (in years, on his last birthday).

The definition of a mapping does not require that two distinct elements x, x' of the domain should have distinct images. In our example, we have $\theta(x) = \theta(x)$ whenever x, x are two people of the same age.

Also we do not require that every element y of the range should be the image of some element x of the domain. In the same example, if we take $y = -3$, then there is no x in P such that $\theta(x) = y$. However it is useful to have special names for those mappings which do satisfies these special requirements.

A mapping $\theta : A \longrightarrow B$ is *injective* if whenever x, x are distinct elements of A, then $\theta(x)$, $\theta(x)$ are distinct elements of B. A mapping $\theta : A \longrightarrow B$ is *surjective* if for each element y of B there is at least one element x of A such that $\theta(x) = y$. A mapping $\theta : A \longrightarrow B$ is *bijective* if it is both injective and surjective. It is sometimes called a *bijection*.

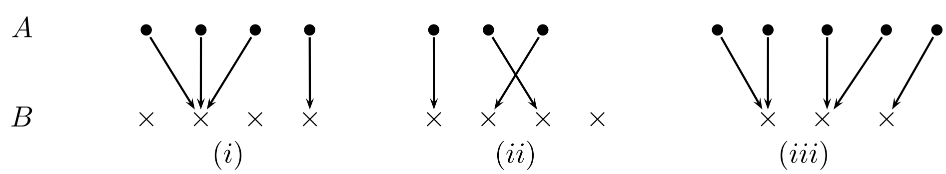

Figure 1

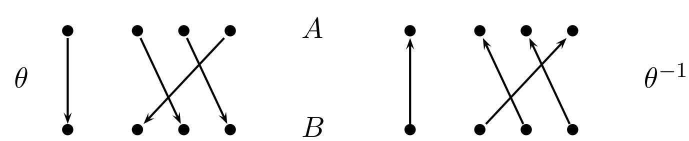

Figure 2

Adapted from *Sets and groups*, by J.A. Green.

3 Questions

1. Three mappings are shown in figure 1, (in each case the row of dots is A and the row of crosses is B). Say in each case whether it is injective and/or surjective.
2. Give an example of a function $f : \{-1; 0; 1\} \longrightarrow \{-1; 0; 1\}$ which is not injective.
3. Give an example of a function $f : \{-1; 0; 1\} \longrightarrow \{-1; 0; 1\}$ which is not surjective.
4. What kind of mapping is represented on figure 2 ? What does θ^{-1} mean ?
5. Which theorem can prove that a real function is a bijection ?
6. If $\theta : A \longrightarrow B$ is injective such that $\theta(x) = \theta(x')$, where x, x' are two elements of A, what can you say about x and x' ?
7. If f and g are two injective functions such that $g \circ f$ exists, prove that $g \circ f$ is injective.