Épreuve de section européenne

Mersenne primes

In mathematics, a Mersenne prime is a prime number that is one less than a prime power of two. For example, 31 is a Mersenne prime and so is 7. On the other hand, $2047 = 2^{11} - 1$, for example, is not a prime, because although 11 is a prime (making it a candidate for being a Mersenne prime), 2047 is not prime. Throughout modern times, the largest known prime number has very often been a Mersenne prime.

More generally, Mersenne numbers (not necessarily primes, but candidates for primes) are numbers that are one less than a prime power of two ; hence, $M_n = 2n - 1$. Most sources restrict the term Mersenne number to where n is prime as all Mersenne primes must be of this form as seen below.

Mersenne primes have a close connection to perfect numbers, which are numbers that are equal to the sum of their proper divisors. Historically, the study of Mersenne primes was motivated by this connection. In the 4th century BC Euclid demonstrated that if M is a Mersenne prime then $\frac{M(M+1)}{2}$ is a perfect number. Two millennia later, in the 18th century, Euler proved that all even perfect numbers have this form. No odd perfect numbers are known, and it is suspected that none exists.

It is currently unknown whether there is an infinite number of Mersenne primes.

Adapted from *Wikipedia, the free encyclopedia.*

Questions

1. Prove that 31 and 7 are Mersenne primes.
2. Prove that 2047 is not prime.
3. Prove that if n is even and greater than 3, M_n is not prime.
4. What are the proper divisors of an integer ?
5. What is a perfect number ?
6. Find one perfect number less than 20.
7. What is the perfect number associated to the Mersenne prime 7 ? Check that is is indeed perfect.
8. Prove that if M is a Mersenne prime then $\frac{M(M+1)}{2}$ is a perfect number.