

Épreuve de section européenne

We're getting the hang of prime numbers

Most children go through a phase of playing “I bet I can think of a bigger number than you can”. It comes just after they realize there is no such thing as the biggest number in the world, so that you can always count to a higher number than any your friend can name, however high that may be. All this is a bit obvious. But some of the best mathematicians in world history have grappled with an almost equally simple question, which is “can you think of a higher prime number than I can?”

A prime number is any number greater than 1 that can only be divided by 1 and itself. The first few prime numbers are easy to recognize.

It is very difficult to find the highest known prime, because there is no quick way of knowing whether any large number can be broken down into other divisors. If it can, it is not prime, but you cannot be sure unless you find a divisor.

Fortunately, there is one short cut, which is to use the particular sub-group of prime numbers first listed by Marin Mersenne, a 17th century French mathematician. (Well actually, he tried to list them, but made some mistakes that have been overlooked by history.) Mersenne primes are all of the form, 2 raised to the power n , less 1. For example, 2 raised to the power of 3, less 1 is 7, and 7 is a prime number. So 7 is a Mersenne prime.

In the ancient world, they only knew about four Mersenne primes [...]. The next one in the sequence, 8191, was only discovered in 1456, and only eight more were discovered until 1952. But then computer power started to take over. [...] Since 1952, another 30 Mersenne primes have been discovered, about three times the number that had been discovered in all of human history before then. The largest currently known, in case you want to impress your friends, is 2 raised to the power of 30,402,457 (less 1, of course)

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Questions

1. What is the definition of a prime number which is given in the text?
2. List the first few prime numbers.
3. Who was Mersenne? Are all the numbers of the form “2 raised to the power n , less 1” prime numbers? List all the Mersenne primes less than 150.
4. Explain the “of course” in the last sentence.
5. Why is it so difficult to check whether an integer is a prime number?
6. Using the decimal logarithm function, can you tell how many digits we need to write the largest currently known prime?