Épreuve de section européenne

Can two different numbers be the same ?

My 16-year-old daughter came home from her GCSE maths course the other day to pose an important question. Is 0.9 recurring equal to 1 ? Certainly not, I said, assuming that something had gone seriously awry with the standard of maths teaching in the London comprehensive system. But I was completely wrong.

For those readers who went missing sometime before their own GCSE in maths, 0.9 recurring means 0.99999. . . with an infinite number of further 9s added on the end. Every time an extra 9 is added, you get closer to 1, but you never quite get there. So, it seemed clear to me that 0.9 recurring could not be equal to 1, though it would approach 1 in the limit of the recurring series.

In fact, though, it seems that most number theorists would argue that both numbers are exactly the same. There are many sophisticated proofs of this proposition, but you may prefer the following intuitive proof. One third, written as a decimal, is 0.3 recurring. Now multiply this by 3, and you get 0.9 recurring. But 3 times one third can only be equal to 1. Therefore 0.9 recurring must be equal to 1.

I think that one reason why people, including myself, find this proposition so hard to swallow is that human beings have a lot of trouble with the meaning of infinity.

Adapted from *The Guardian, June 22, 2006.*

Questions

1. Explain the expression “0.9 recurring”.

2. Another proof : let x be equal to 0.9 recurring. What is $10x$? Write $10x$ as a function of x. Complete the proof.

3. State two theorems you know involving the notion of infinity.

4. What is a geometric sequence ?

5. Let n be a nonzero natural integer, and u_n be equal to 9×10^{-n}. What is u_1, u_2, $u_1 + u_2$? Using the sum of the terms of the sequence, prove that 0.9 recurring is equal to 1.