Épreuve de section européenne

Geodesic domes

For any solid figure with polygons for faces, Euler's formula, $F+V-E=2$, must hold. In this formula, F, V and E stand for the number of faces, vertices, and edges, respectively, that the figure has.

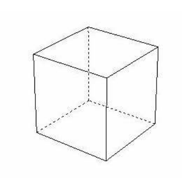

Figure 1.

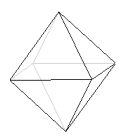

Figure 2.

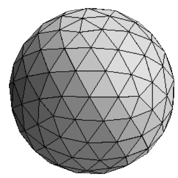

Figure 3.

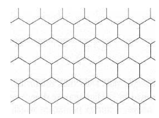

Figure 4.

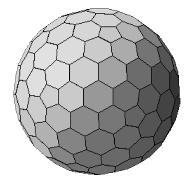

Figure 5.

A spherical dome is an efficient way of enclosing space, since a sphere holds a greater volume than any other container with the same surface area. In 1947, R. Buckminster Fuler patented the geodesic dome, a framework made by joining straight pieces of steel or aluminium tubing in a network of triangles. A thin cover of aluminium or plastic is attached to the tubing (fig. 3).

Although a grid of hexagons will interlock nicely to cover the plane (fig. 4), they cannot interlock to cover a sphere unless twelve of the hexagons are changed to pentagons (fig. 5).

Adapted from *Geometry*, by Jurgensen/Brown and various sources on the internet.

Questions

1. In the plane, what is a polygon ? Give a few examples.

2. Verify Euler's formula for a cube (fig. 1) and then for an octahedron (fig. 2).

3. What is the definition of a sphere ?

4. Let us use an indirect proof and assume that hexagons can interlock to cover a sphere. Let's assume that the framework has n faces, all hexagons. Thus $F=n$.

- **a.** To find V, the number of vertices on the framework, notice that each hexagon contributes 6 vertices, but each vertex is shared by 3 hexagons. What is V as a function of n ?
- **b.** To find E, the number of edges of the framework, notice that each hexagon contributes 6 edges, but each edge is shared by 2 hexagons. What is E as a function of n ?
- **c.** According to Euler's formula, $F+V-E$ must equal 2. Does it ?
- **d.** What does this contradiction tell you ?

5. Suppose now that 12 of the n faces of the framework are pentagons. Show that $V=\frac{6n-12}{3}$ and that $E=\frac{6n-12}{2}$. Then calculate $F+V-E$. Does this agree with the text ?