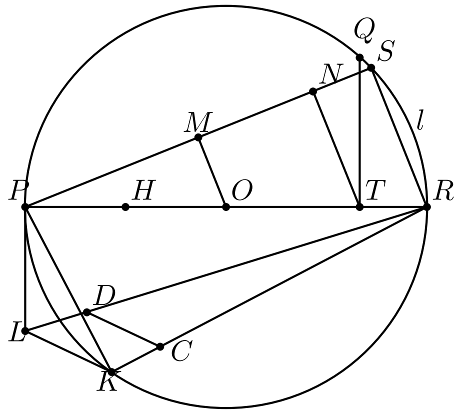
Épreuve de section européenne

Squaring the circle



Let PQR be a circle with centre O, of which a diameter is PR. Bisect PO at H and let T be the point of trisection of OR nearer R. Draw TQ perpendicular to PR and place the chord $RS = TQ$.

Join PS, and draw OM and TN parallel to RS, with M and N on RS. Place a chord $PK = PM$, and draw the tangent $PL = MN$. Join RL, RK and KL. Place C on RK such that $RC = RH$. Draw CD parallel to KL, meeting RL at D.

Then the square on RD will be equal to the circle PQR approximately.

For

$$RS^2 = \frac{5}{26}d^2$$

where d is the diameter of the circle. Therefore

$$PS^2 = \frac{31}{26}d^2.$$

But PL and PK are equal to MN and PM respectively. Therefore

$$PK^2 = \frac{31}{144}d^2 \text{ and } PL^2 = \frac{31}{324}d^2.$$

Hence

$$RK^2 = PR^2 - PK^2 = \frac{113}{144}d^2 \text{ and } RL^2 = PR^2 + PL^2 = \frac{355}{324}d^2.$$

But

$$\frac{RK}{RL} = \frac{RC}{RD} = \frac{3}{2}\sqrt{\frac{113}{355}}$$

and $RC = \dfrac{3}{4}d$. Therefore

$$RD = \frac{d}{2}\sqrt{\frac{355}{113}} = r\sqrt{\pi},$$

very nearly.

Note.-If the area of the circle be 140,000 square miles, then RD is greater than the true length by about an inch.

From *Journal of the Indian Mathematical Society*, 1913, by Srinivasa Ramanujan

Questions

1. Explain each step of the proof.

2. Is this construction exact ? If not so, what information is given in the text about its precision ?