Épreuve de section européenne

Laws of the algebra of sets

The union of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to A or to B. (here "or" is used in the sense of and/or).

The intersection of two sets A and B, denoted by $A \cap B$, is the set of all elements which belong to both A and B.

The complement of a set A, denoted by $\overline{A}$, is the set of elements which do not belong to A.

We discuss two methods of proving equalities involving set operations. The first is to describe what it means for an object x to be an element of each side, and the second is to use Venn diagrams. For example consider the first of De Morgan's laws : $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$.

Method 1

We first show that $\overline{(A \cup B)} \subset \overline{A} \cap \overline{B}$.

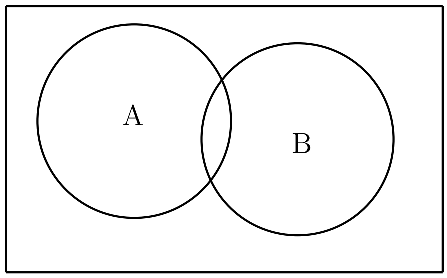
If $x \in \overline{(A \cup B)}$, then $x \notin A \cup B$. Thus $x \notin A$ and $x \notin B$, so $x \in \overline{A}$ and $x \in \overline{B}$, hence $x \in \overline{A} \cap \overline{B}$.

Next we show that $\overline{A} \cap \overline{B} \subset \overline{(A \cup B)}$.

Let $x \in \overline{A} \cap \overline{B}$. Then $x \in \overline{A}$ and $x \in \overline{B}$, so $x \notin A$ and $x \notin B$. Hence $x \notin A \cup B$, so $x \in \overline{(A \cup B)}$.

We have proven that every element of $\overline{(A \cup B)}$ belongs to $\overline{A} \cap \overline{B}$ and that every element of $\overline{A} \cap \overline{B}$ belongs to $\overline{(A \cup B)}$. Together these inclusions prove that the sets have the same elements, i.e., that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$.

Method 2



Adapted from *Finite Mathematics*. S.Lipschutz, J. Schiller.m

Questions

1. Using a Venn diagram, explain what the intersection, union and complement of sets are.

2. **a.** What general method do we use in method 1 to prove that two sets are equal ?

b. Explain this proof in your own words.

c. Use a Venn diagram to explain this De Morgan equality.

3. Try to prove that $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$ using a similar method.

4. If you have the time to do so, try to prove that $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$ using one of the two methods. Can you think of another distributive law ?