Épreuve de section européenne

Injections, surjections and bijections

In mathematics, injections, surjections and bijections are classes of functions distinguished by the manner in which arguments (input expressions from the domain) and images (output expressions from the codomain) are related or mapped to each other.

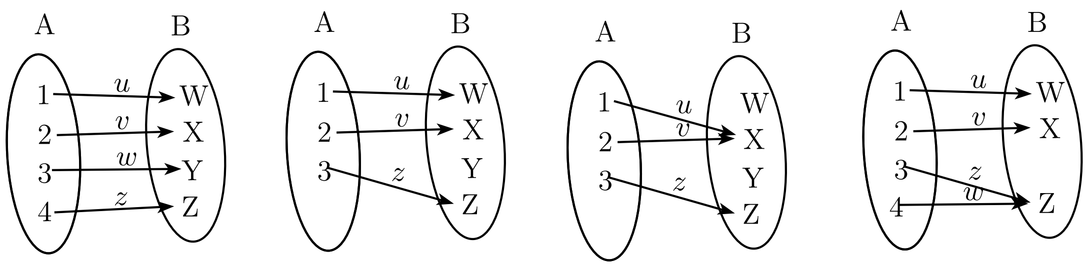
A function $f : A \to B$ is injective (or one-to-one) if for any $x_1, x_2 \in A$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$ or, equivalently, if $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$.
One could also say that elements of the codomain are mapped to by at most one element of the domain ; not every element of the codomain, however, needs to have an argument mapped to it. An injective function is an injection.

A function is surjective (or onto) if every element of the codomain is mapped to by some element (argument) of the domain ; this is expressed logically by saying that, for any $y \in B$, there exists $x \in A$ such that $y = f(x)$.
Note that with this definition, some images may be mapped to by more than one argument. A surjective function is a surjection.

A function is bijective (one-to-one and onto) if and only if it is both injective and surjective. (Equivalently, every element of the codomain is mapped to by exactly one element of the domain.) A bijective function is a bijection (one-to-one correspondence).

An injective function needs not be surjective (not all elements of the codomain may be associated with arguments), and a surjective function needs not be injective (some images may be associated with more than one argument). The four possible combinations of injective and surjective features are illustrated in the following diagrams.

- Non-injective and non-surjective.
- Injective and surjective (bijective).
- Non-injective and surjective.
- Injective and non-surjective.



Adapted from *Wikipedia.org*.

Questions

1. Associate each diagram with the corresponding "combination of injective and surjective". (Which is which ?)
2. Are the exponential and the natural logarithm functions injective, surjective and/or bijective ?
3. What theorem that you studied this year can help you prove that a real valued function is a bijection on a given interval ?
4. Let f be the function defined by $f : \mathbf{R} \to \mathbf{R} : x \mapsto |x|$. Is f injective ? Is f surjective ?
5. Give examples of real valued functions corresponding to each of the four cases of the first question.