Épreuve de section européenne

Angle trisection

Problems as Angle trisection (i.e. trisect an arbitrary angle) are legendary not because they did not have solutions. No, numerous simple solutions have been found yet by Greek mathematicians. The problem was in that all known solutions violated an important condition for this kind of problems, one condition imposed by the Greek mathematicians themselves : Valid solutions to the construction problems are assumed to consist of a finite number of steps of two kinds only : drawing a straight line through two points with a ruler (or rather a straightedge as no marks are allowed on the ruler) and drawing a circle with a given center and radius.

That there may be a difficulty trisecting an angle is rather surprising. The problem of trisecting a line segment is no more difficult than finding its n-th part for an arbitrary n. However, the general problem of trisecting an angle (i.e., trisecting an arbitrary angle) is not solvable in a finite number of steps. Two points must be made.

Firstly, some angles are trisectable with a ruler and a compass. An obvious example is supplied by the three quarters of the right angle : 67.5°. (The angle itself is constructible as it is obtained by two consecutive angle bisections. Its third is obtained along the way.) Angles of 30° (draw a right triangle with a side 1 and hypotenuse 2) and 45° (bisect the right angle) are both constructible. Therefore, the latter also admits a classical trisection. However, and this is where the difficulty lies, an arbitrary angle can't be trisected with a ruler and a compass, as stated above.

Secondly, consider the geometric series $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$ that adds up to $\frac{1}{3}$. Once we know how to bisect an angle, we may also find its 2^n-th part, for any n. In particular, one may construct $\frac{1}{4}$, $\frac{1}{16}$,. . .of any angle and, in principle, find its third after an infinite number of steps. This solution is universal but requires an infinite number of steps, which is forbidden here.

The problem was settled in 1837 by Pierre Laurent Wantzel (1814-1848) who proved that there was no way to trisect a 60° angle in the classical framework.

From

Questions

1. Which condition was imposed by the Greek mathematicians to trisect an angle ?
2. How do you bisect an angle using a straightedge and a compass ?
3. Explain the "obvious example" of 67.5° : Why the angle itself is constructible ? What is the measure of its third ? How is it "obtained along the way" ?
4. How can you construct angles of 45° and 30° ?
5. Prove that the geometric series $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$ adds up to $\frac{1}{3}$. How can it be used to trisect an angle ? Why is this however not a solution to the original problem ?