Épreuve de section européenne

The Amazing Number 1089

Choose any three-digit number (where the units and hundreds digits are not the same). Reverse the digits of this number you have selected. Subtract the two numbers (naturally, the larger minus the smaller). Once again, reverse the digits of this difference. Now, add your last two numbers. We say that whatever the initial number, the result of this computation will be 1089.

Is it true ? Well, you could try all possible three-digit numbers to see if it works. That would be tedious and not particularly elegant. The beginning of a much more elegant proof is given below.

We shall represent the arbitrarily selected three-digit number, $\overline{htu}$, as $100h + 10t + u$.

Let $h > u$, which would be the case in either the number you selected or the reverse of it. In the subtraction, $u - h < 0$; therefore, take 1 from the tens place (of the minuend[1]), making the units place $10 + u$.

Since the tens digits of the two numbers to be subtracted are equal, and 1 was taken from the tens digit of the minuend, then the value of this digit is $10(t - 1)$. The hundreds digit of the minuend is $h - 1$, because 1 was taken away to enable subtraction in the tens place, making the value of the tens digit $10t - 1 + 100 = 10(t + 9)$.

We can now do the first subtraction :

$$[100(h - 1) + 10(t + 9) + (u + 10)] - [100u + 10t + h].$$

Adapted from *Maths Wonders to Inspire Teachers and Students* by Alfred S. Posamentier

Questions

1. Carry out the computations with the number 528.
2. How many numbers would you need to check if you wanted to check all three-digit numbers ?
3. Explain the choice of letters h, t and u.
4. **a.** Explain why, when subtracting $100u+10t+h$ from $100h+10t+u$, we need to “borrow” a ten and a hundred.

   b. Explain why the tens digit of the first operation is always 9.
5. Complete the proof.

[1]In a subtraction $a - b$, the number a is the minuend.