

Épreuve de section européenne

Ruth-Aaron pairs

On the warm and cloudy evening of April 8, 1974, Dodger lefthander Al Downing threw a high fastball into Hank Aaron's ample strike zone. At 9 : 07 P.M. Aaron whipped his bat and sent the ball arcing deep into left centerfield all the way over the fence. The Atlanta hometown crowd of 53775 roared as Aaron's 715th home run eclipsed Babe Ruth's 1935 record of 714. 'I just thank God it's all over,' said Aaron, expressing the same kind of relief a mathematician might feel after doggedly solving a forty-year-old problem. [...]

Pomerance was a young assistant professor then and he noticed that the product of 714 and 715 was also the product of the first seven primes [...]. A student of one of Pomerance's colleagues also found another interesting property of 714 and 715 : the sum of the prime factors of 714 equaled the sum of the prime factors of 715. [...].

Pomerance called pairs of consecutive integers that had this property *Ruth-Aaron pairs*. Such pairs were extremely rare.

Adapted from *The Man Who Loved Only Numbers* by Paul Hoffman

Questions

1. The first paragraph of this text is about an event that happened during a baseball match between the Los Angeles Dodgers and the Milwaukee and Atlanta Braves. In baseball, a *home run* is typically achieved by hitting the ball over the outfield fence without first touching the ground or outfield fence. Explain in a few words what special event is told.
2. Check that the product of 714 and 715 is also the product of the first seven primes.
3. One possible definition is to count only distinct prime factors.
 - a. Prove that in that case, (49, 50) is a Ruth-Aaron pair.
 - b. Find two such pairs lower than (49, 50).
4. Another possible definition is to count repeated prime factors as many times as they appear.
 - a. Which ones of the pairs you found in the previous questions are still Ruth-Aaron pairs with this definition?
 - b. Prove that in that case, (77, 78) is a Ruth Aaron pair.
 - c. Find two news pairs lower than (49, 50).