Épreuve de section européenne

Non-transitive dice

The game involves two players, each rolling a die, and the one with the higher number wins. Each player starts by choosing a die from a selection of three, labelled A, B and C. Curiously, over the course of several throws, die A beats die B, while B beats C, while C beats A !

Common sense tells us that die C should not beat die A, because we have a notion that if A is better than B, and B is better than C, then A should be better than C. Mathematicians call this usual relationship transitive. Consequently, the dice that defy this rule [...] are called non-transitive dice.

Here are the faces for one possible set of three non-transitive dice. Each die has three numbers, each one repeated to cover the six faces.

Die A - 3, 3, 5, 5, 7, 7

Die B - 2, 2, 4, 4, 9, 9

Die C - 1, 1, 6, 6, 8, 8

So what happens if you play die A against die B ? On average A beats B 5/9 of the time.

Similarly, we can analyse die B played against die C [and see that] on average B beats C 5/9 of the time.

Finally, we can play die C against die A and see also that on average C beats A 5/9 of the time.

Even better, Allen J Schwenk of Western Michigan University discovered a set of three non-transitive dice that exhibit a very peculiar (and useful) property. The dice have the following faces :

Die A - 1, 1, 1, 13, 13, 13

Die B - 0, 3, 3, 12, 12, 12

Die C - 2, 2, 2, 11, 11, 14

First, as before, if your opponent picks any die, then you can always pick one that beats it. However, if you are forced to pick first and your opponent happens to then pick the better die, then a slight rule change still gives you the edge. Just play the game such that each die is rolled twice and it is the highest total that wins. Bizarrely, your inferior die suddenly becomes superior.

From *How to pick a winning hand every time*, guardian.co.uk, by Simon Singh

Questions

1. An omitted paragraph of this article refers to the famous game of RPS, in which the two players show their hands in one of three ways : rock (clenched fist), paper (open, flat hand), or scissors (forefinger and middle finger form a 'V'). Explain how this game is related to non-transitive dice.
2. Check the probabilities given for the first set of dice.
3. Compute the probabilities for the second set of dice, when throwing the die only once.
4. Compute the probabilities for the second set of dice, when throwing the die twice, and check the last sentence in the document.