Épreuve de section européenne

Urban geometry

You have just arrived in town at the central railroad station and you are hoping to be able to make the 8 o'clock performance of the opera. The opera house is located at a point which, if we think of the railroad station as being at $(0, 0)$, has coordinates $(5, 12)$.

Traffic is so heavy in town you estimate you can actually walk as fast as a taxi can drive you there. How far is the opera house from the train station ?

Since you are at $(0, 0)$ and have to get to $(5, 12)$, you fall back on your mathematical skills. You use your knowledge of Euclidean geometry and of the Pythagorean Theorem to see how far the opera house is from where you are located. [...] The opera house's distance from the train station would be 13 units. However, unfortunately in an urban environment one can not typically get from one place to another the way a crow or helicopter can. The diagram shows a schematic for a piece of a typical urban center such as a part of Manhattan.

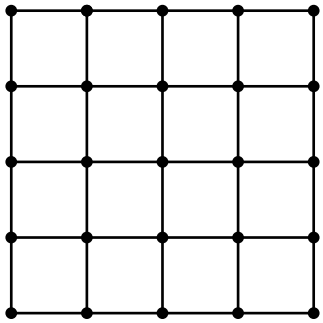

Figure 1 (A grid graph representing the layout of a piece of a street network of a city)

To get from one corner to another, one has to move along the line segments shown in the diagram which are all assumed to have the same length 1. We can also introduce a coordinate system into this diagram by labeling the lower left hand dot with $(0, 0)$ and the upper right hand dot would then have the coordinates $(4, 4)$, The distance in this situation, between $(1, 2)$ and $(4, 4)$, would be 5. Note, however, that the distance between $(1/4, 0)$ and $(7/8, 1)$ would not be $(7/8 - 1/4) + (1 - 0)$ but the minimum of $1/8 + 1 + 3/4$ and $7/8 + 1 + 1/4$.

From *Taxi !*, an AMS Feature Column, by Joe Malkevitch

Questions

1. Check that the straight-line distance between the station and the opera is indeed 13.
2. Explain why "the distance between $(1/4, 0)$ and $(7/8, 1)$ would not be $(7/8 - 1/4) + (1 - 0)$ but the minimum of $1/8 + 1 + 3/4$ and $7/8 + 1 + 1/4$".
3. Find out the urban distance between $(3/2, 1)$ and $(7/2, 4)$.
4. When is the urban distance equal to the straight-line distance ?
5. Consider the points $A(1, 2)$ and $B(2, 1)$.
   a. Using the straight-line distance, find out all points on the graph equidistant from A and B.
   b. Find out the urban distance between A and $C(4, 2)$, and the urban distance between A and C.
   c. Using the urban distance, find out all points on the graph equidistant from A and B.