Épreuve de section européenne

Euler's straight line

In all triangles, the center of the circumcircle, the point of intersection of the medians, and the point of intersection of the altitudes are situated in this order in a straight line – the Euler line – and are spaced in such a manner that the altitude intersection is twice as far from the median intersection as the center of the circumcircle is.

Leonhard Euler (1707 – 1783) was one of the greatest and most fertile mathematicians of all time. His writings comprise 45 volumes and over 700 papers, most of them long ones, published in periodicals. The above theorem is among the results of one of these papers which appeared in the journal Novi commentarii Academiae Petropolitanae in 1765. The following proof of Euler's theorem is distinguished by its great simplicity.

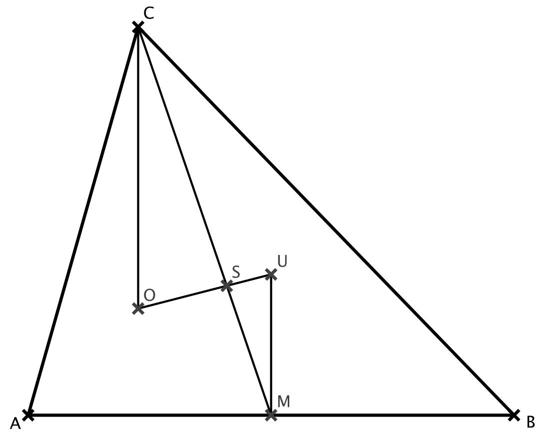
In the triangle ABC, let M be the midpoint of side AB, S the median intersection, which lies on CM, so that

$$SC = 2SM \qquad (1)$$

and U the center of the circumcircle, lying on the perpendicular bisector of AB.

We extend US by SO so that $SO = 2SU$ (2) and join O to C.

According to (1) and (2) the triangles MUS and COS are similar. Consequently, $CO \parallel MU$, that is $CO \perp AB$, or expressed verbally, the line connecting the point O with a vertex of the triangle is perpendicular to the side of the triangle opposite the vertex ; consequently, the connecting line is an altitude of the triangle.



The three altitudes consequently pass through point O. This is, therefore, the altitude intersection, and Euler's theorem is proved.

Adapted from *100 Great problems of elementary mathematics* by Heinrich Dörrie, Dover, 1965

Questions

1. Knowing that Euler's first paper was published in 1726, what is the average number of papers Euler published every year in his life ?
2. The name of the journal in which the mentioned paper appeared is in Latin. Do you know the modern name of the Russian town Petropolis ?
3. Justify equality (1), the similarity of triangles MUS and COS, the parallelism of CO and MU.