Épreuve de section européenne

An exponential model for Learning

Psychologists have found that in many learning situations a person's rate of learning is rapid at first and then slows down. Finally, as the task is mastered, the person's level of performance reaches a level above which it is almost physically impossible to rise. For example, within reasonable limits, each person seems to have a certain maximum capacity for memorizing a list of nonsense syllables.

Suppose that a subject can memorize M syllables in a row if given sufficient time, say one hour, to study the list but cannot memorize $M+1$ syllables in a row even if allowed several hours of study. By experimentation, the psychologist can determine an empirical relationship between the number of nonsense syllables memorized accurately and the number of minutes of study time. It turns out that a good model for this situation is given the function N defined below

$$N(t) = M(1 - e^{-kt}) \tag{1}$$

and the curve of this function is named the learning curve.

When t is close to zero, e^{-kt} is close to one and the number of syllables is small. As t increases, e^{-kt} becomes small and so $v(t)$ approaches M.

We can show that the number of syllables given in (1) satisfies the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = k[M - N(t)], \text{ with } N(0) = 0. \tag{2}$$

The differential equation (2) says that the rate of change in N is proportional to the difference between the terminal velocity M and the actual velocity N. It is not difficult to show that the only solution of (2) is given by the formula in (1). The two equations (1) and (2) arise as mathematical models in a variety of situations.

The slope of the learning curve at a time t is approximately the number of additional syllables that can be memorized if the subject is given one more minute of study time. Thus the slope is a measure of the rate of learning.

Adapted from *Calculus and its applications*
by Goldstein, Lay & Schneider, Prentice-Hall, NJ, 1980

Questions

1. Why do psychologists need lists of *nonsense* syllables for their experiments on learning ?
2. Draw the Learning curve using equation (1) as a definition (taking $M = 50$ and $k = 1$). Prove the variations and limits suggested by the text ?
3. Prove that the function $N(t)$ given in (1) actually satisfies the differential equation (2), with its initial condition.
4. Explain the last paragraph of the text (you can use one of the definitions of the derivative).