Épreuve de section européenne

The problem of antidifferentiation

The elementary functions may all be differentiated by elementary rules, the process yielding elementary functions in turn. Antidifferentiation succeeds *by definition* for the derivatives of each of the elementary functions. This circle is closed. But given an elementary function, does it follow that its antiderivative must be elementary in turn ?

No, unfortunately. There are plenty of elementary functions that fail of antidifferentiation if the standard of success is that the revealed antiderivative be an elementary function. The functions

$$\frac{e^t}{t}, \frac{\sin t}{t}, \frac{1}{\ln t},$$

and many others resist antidifferentiation in elementary terms. There is *no* elementary function $f(t)$ whose derivative is $\frac{\sin t}{t}$. This is not a mystery ; it is just the way things are. A circle closed in one respect, differentiation, is open in another, antidifferentiation. The hope that antidifferentiation might return an elementary function to an elementary function is a part of a system of childish illusions that dominates elementary mathematics.

Adapted from *A Tour of the Calculus* by David Berlinski

Questions

1. Explain the words *derivative* and *antiderivative*.
2. Give a few examples, not only powers, of simple functions with simple antiderivatives.
3. Explain the words *by definition* in the second sentence.
4. Differentiate the three functions given in the text.
5. Find an antiderivative of the function te^t.