Épreuve de section européenne

Mantissae

"Mantissae" probably seems as archaic to today's readers as a starter crank on the front of an automobile, but until 1960 or so every high-school science student was taught the lore of logarithms, and in particular how to use "common" (base-10) logarithmic tables in calculation. Their use involved the separation of a logarithm into two parts : its integer part (the characteristic) and its fractional part (the mantissa). Here is an example :

Suppose, before the days of hand-held calculators, you needed a rapid way to multiply four-digit numbers, and to divide that product by another four-digit number, with an answer accurate to three digits. Say

$$\frac{86.73 \times 1.265 \times 7607}{.3018}.$$

Procedure : You think of each of the numbers as a power of 10 times a number between 1 and 10 :

$$86.73 = 10^1 \times 8.673\,;\ 1.265 = 10^0 \times 1.265\,;\ 7607 = 10^3 \times 7.607 \text{ and } .3108 = 10^{-1} \times 3.108.$$

When you take logarithms, since $\log(ab) = \log a + \log b$,

$$\log(86.73) = 1 + \log(8.673)\,;\ \log(1.265) = 0 + \log(1.265)\,;$$

$$\log(7607) = 3 + \log(7.607) \text{ and } \log(.3018) = -1 + \log(3.108).$$

The second term in each of the logs is a number between 0 and 1 : this will be the mantissa ; the leading term is the characteristic. To obtain the log of the answer that we want, $\log(86.73) + \log(1.265) + \log(7607) - \log(.3018)$, we make two calculations. First we add or subtract the characteristics ; this is an integer calculation. In this case the total is 5. Then you consult a four-place logarithmic table for the mantissae :

$$\log(8.673) = .93817\,;\ \log(1.265) = .10209\,;\ \log(7.607) = .88121 \text{ and } \log(3.018) = .47972.$$

The mantissae total (with signs) to 1.44175. You chop off the "1" and add it to the characteristic. The log table gives $\log(2.765) = .44170$ and $\log(2.766) = .44185$. Since you only expect 3 places of accuracy, you can take 2.765 as the mantissa contribution to the product, which you calculate as $10^{5+1} \times 2.765 = 2765000$.

From *Simon Newcomb and "Natural Numbers"*, http ://www.ams.org by Tony Phillips

Questions

1. Why were logarithmic tables ever needed, and what information did they show ?
2. Explain precisely what the characteristics and the mantissae are in this method.
3. Explain why the result for the characteristics is 5.
4. Use the properties of the logarithm and the example to show that the method is correct.
5. Use the method to get an approximate value of $\log(7629 \times 52.41)$. The values for the mantissae shall be given by the calculator, with 5DP.