## Épreuve de section européenne

## Let's make a deal

Monty Hall Game Show *Let's Make A Deal* took place during the seventies. The particular game that we are concerned with here is when Monty Hall offers you the opportunity to win what is behind one of three doors. Typically there is a really nice prize (i.e., a car) behind one of the doors and a not-so-nice prize (i.e., a goat) behind the other two. After selecting a door, Monty proceeds to open one of the doors you have not selected. It is important to note here that Monty does NOT open the door that conceals the car. At this point, he asks you if you want to switch to the other door before revealing what you have won.

The intuition of most people tells them that each of the doors, the chosen door and the unchosen doors, are equally likely to contain the prize so that there is a 50-50 chance of winning with either selection. This, however, is not the case.

This problem named The Monty Hall Paradox was discussed in the popular "Ask Marylin" questionand-answer column of the Parade magazine.

Marilyn's answer was that the contestant should switch doors and she received nearly 10,000 responses from readers, most of them disagreeing with her. Several were from mathematicians and scientists. Eventually, she issued a call to Maths teachers among her readers to organize experiments. Some readers with access to computers ran computer simulations. At last, the truth was established and accepted.

In fact the truth proved to be more complex. With the accumulated experience, it is safe to assert that the major thrust of the controversy was not directed at Marylin's solution but at her interpretation of the formulation. Indeed, there are two ways to interpret Monty's behaviour as described, "...It is important to note here that Monty would NOT open the door that concealed the car".

One interpretation is that he just opens the door behind which, apparently by pure chance, a goat is to be found. We must decide what it means if Monty should happen to open the door with the car behind by accident. The problem only says that Monty opened a door with a goat behind it, so we interpret this as such if the car is revealed then the game is over and the next contestant plays the game.

Marylin, on the other hand, chose another interpretation. She sensed that there is a reason for the host to know the location of the items behind the doors as it has been mentioned explicitly. The problem was that, although it was not followed by an equally explicit statement, the host, basing his choice on his knowledge, **always** opens the door to reveal a goat.

Adapted from Interactive Mathematics Miscellany and Puzzles and from the official website of the University of California, San Diego

## Questions

- 1. Explain the game and the dilemma with your own words.
- **2.** Explain the controversy that followed.
- **3.** Make two tree diagrams of the game in the "Monty knows version", one in the switching scenario and one in the non-switching scenario. What is the probability of winning?
- 4. Do the same in the "Monty does not know version". What is the probability of winning?
- **5.** Suppose you were the contestant, ignoring whether Monty knows or not, what would your decision be?