Épreuve de section européenne

A magical multiplying method

I well remember the wonder that I experienced when I learned about logarithms. The thing that most caught my attention was that you could "multiply two numbers" through a not-so-simple addition process... Of course, this is just a "taste" of a much larger and grander topic in Mathematics, but it is meant to give a connection to what I want to show you this time, namely how you can "multiply by adding", using a much simpler, arithmetic basis.

Before we can proceed, we have to introduce triangular Numbers. 10 is a triangular number, because 10 things can be arranged in a triangular array like this :

★
★ ★
★ ★ ★
★ ★ ★ ★

From this sort of picture it is easy to form and determine many other triangular numbers.

Here we see that 1, 3, and 6 are the first three triangular numbers T_1, T_2 and T_3. And of course with these few examples we can see a shortcut for finding other triangular numbers.

$$\begin{array}{rcrcr} T4 & = & 10 & = & 1+2+3+4 \\ T3 & = & 6 & = & 1+2+3 \\ T2 & = & 3 & = & 1+2 \\ T1 & = & 1 & = & 1 \end{array}$$

Now we're ready to show you the magical way to multiply without multiplying anything. I call it the "Magical Multiplying Method" (or the "3M" way). First, I must confess we will subtract a little too. Second, you will need a table containing the values of triangular numbers.

Let's take as an example 15×9 :

1. Take the larger factor 15 and find T_{15} in the table mentioned above.
2. Subtract 1 from 9, the smaller factor, getting 8. Find T_8 in the table.
3. Subtract the two factors, $15 - 9$; that's 6. Find T_6.
4. Add the results of Steps 1 & 2, then subtract the result from Step 3. That's your product !

Well, I never said it was going to be easier, shorter or anything like that.

Adapted from Terry Trotter website www.trottermath.net

Questions

1. The first paragraph of the text refers to a famous property of logarithms. Which one ?
2. **a.** Express T_{n+1} in terms of T_n and n.
   b. Use **a.** to give in a table the values of T_n, for n from 1 to 15.
3. **a.** Check the calculation 15×9 in the text.
   b. Compute 13×12 using the "3M" way.
4. Translate into an algebraic formula the "3M" $a \times b$, where a and b are two unequal natural numbers.
5. **a.** Prove that for any natural number n, $T_n = \frac{n(n+1)}{2}$.
   b. Hence, prove the formula given in question 4 to get the product $a \times b$, a being greater than b.
   c. What happens if $a = b$? Adapt the method to square natural numbers.