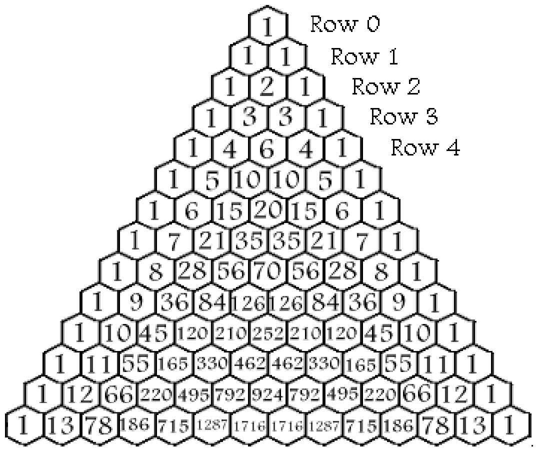
Épreuve de section européenne

APs in Pascal's triangle



Although Pascal's triangle is named after seventeenth century mathematician, Blaise Pascal, several other mathematicians knew about it and applied their knowledge of the triangle hundreds of years before Pascal's birth in 1623. Today, the triangle appears to have been discovered independently by both the Persians and the Chinese during the eleventh century.

Although no longer in existence, the work of Chinese mathematician Chia Hsien showed that he "was using the triangle to extract square and cube roots of numbers". Also having a method of extracting roots of numbers, Omar Khayyam (1048 ? – 1113 ?), a Persian mathematician, seemed to have had knowledge of the so-called Pascal's triangle.

In China, after Chia Hsien's discovery of the relationship between extracting roots and the binomial coefficients of the triangle, several Chinese algebraists continued to work on this topic to solve higher degrees equations than cubic ones. Years after it first appeared in Persia and China, the triangle came to be known as Pascal's triangle with Blaise Pascal's completion of *Traité du triangle arithmétique* in 1654.

Making use of the already known array of binomial coefficients, French mathematician Pascal developed many of the triangle's properties and applications within these writings. As one peers deeper into the triangle's numbers, many interesting patterns may be noticed.

From the website of College of Science and Mathematics
Montclair State University, New Jersey

Questions

1. Is it correct to say that Pascal's triangle was invented by Blaise Pascal ?

2. What was Blaise Pascal's personal contribution to the arithmetic triangle ?

3. Give different problems where the use of Pascal's triangle is helpful, by quoting from the text and also from your maths course.

4. For the following questions, it's important to remember that Pascal's triangle is made of binomial coefficients. For example, the row 3 is made of $\binom{3}{0}$, $\binom{3}{1}$, $\binom{3}{2}$ and $\binom{3}{3}$.

- **a.** Show that $\binom{n}{r-1} = \frac{r}{n-r+1}\binom{n}{r}$ and $\binom{n}{r+1} = \frac{n-r}{r+1}\binom{n}{r}$
- **b.** Show that if $\binom{n}{r-1}$, $\binom{n}{r}$, $\binom{n}{r+1}$ form an arithmetic progression (AP), then $n+2 = (n-2r)^2$.

5. Assuming that the previous equation is equivalent to $r = \frac{1}{2}\left(n - \sqrt{n+2}\right)$ or $r = \frac{1}{2}\left(n + \sqrt{n+2}\right)$, find the first row of Pascal's triangle in which three consecutive terms form an AP, and identify those terms.