Épreuve de section européenne

The problem of squaring a quadrilateral

The Greeks were able to construct a square equal in area to any given polygon.

First construction : to construct a square equal to a given parallelogram whose base and altitude are respectively b and h (figure 1-a), we can use the equation $x^2 = bh$ where x is the side of the square (figure 1-b). First, construct the altitude of the given parallelogram. Then a semicircle is constructed with a diameter equal to $b + h$. At E a perpendicular is erected to DF, meeting the semicircle at G ; $EG = x$.

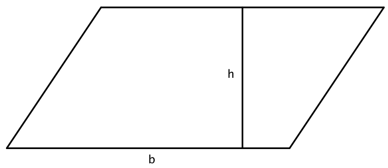

Figure 1-a

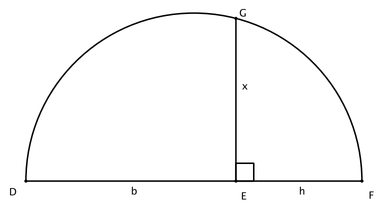

Figure 1-b

Second construction : to construct a square equal to a given quadrilateral, one can proceed as follows : Draw diagonal DB of quadrilateral $ABCD$. Thru[1] C draw a line parallel to DB and intersecting AB extended at F. Draw DF. Then triangle AFD is equal in area to quadrilateral $ABCD$. A square can be constructed equal in area to any given triangle, using the first construction.

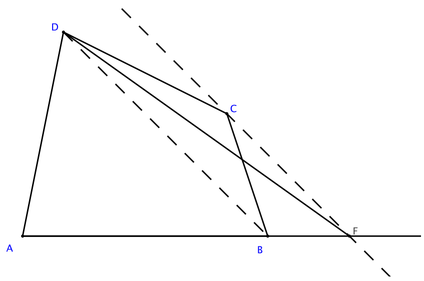

Figure 2

From *Famous Problems of Geometry*, by Benjamin Bold

Questions

1. Proof of the first construction, using figure 1-b :

- **a.** Show that : $\angle GDE = \angle EGF$. Hence deduce that $\frac{h}{x} = \frac{x}{b}$. (Which is equivalent to $x^2 = bh$.)
- **b.** How would you now construct the required square ?
- **c.** How could you next construct a square of area $\frac{1}{2}x^2$ from your previous square ? (Hint : the length of the diagonal of a square is $\sqrt{2}$ times the length of the side).
- **d.** Deduce now a method to construct a square, equal in area to any given triangle whose base and altitude are b and h ?

2. Proof of the second construction, using figure 2 :

- **a.** Why are the areas of triangles DCB and DBF the same ?
- **b.** Hence, prove that the area of quadrilateral $ABCD$ is equal to the area of triangle ADF.
- **c.** How can we now construct a square equal in area to $ABCD$?

1. thru (US) = through (GB)