Épreuve de section européenne

Continued fractions

An expression of the form $a + \dfrac{b}{c + \frac{d}{e + \ldots}}$ is called a *continued fraction*; here the letters a, b, c,... may denote any quantities whatever, but for the present we shall only consider the simpler form $a_1 + \dfrac{1}{a_2 + \frac{1}{a_3 + \ldots}}$, where a_1, a_2, a_3,... are positive integers. This will be usually written in the more compact form

$$a_1 + \frac{1}{a_2 +} + \frac{1}{a_3 +} \ldots$$

To convert a given fraction into a continued fraction :

Let $\frac{m}{n}$ be the given fraction; divide m by n, let a_1 be the quotient and p the remainder; thus

$$\frac{m}{n} = a_1 + \frac{p}{n} = a_1 + \frac{1}{\frac{n}{p}};$$

divide n by p, let a_2 be the quotient and q the remainder; thus

$$\frac{n}{p} = a_2 + \frac{q}{p} = a_2 + \frac{1}{\frac{p}{q}};$$

divide p by q,let a_3 be the quotient and r the remainder; and so on. Thus

$$\frac{m}{n} = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \ldots}} = a_1 + \frac{1}{a_2 +} + \frac{1}{a_3 +} \ldots$$

From *Higher Algebra*, by Hall and Knight, 1964.

Questions

1. What continued fraction is represented by the compact form $1 + \frac{1}{2+} + \frac{1}{3+} + \frac{1}{4}$?
2. Write the continued fraction $1 + \frac{1}{2+} + \frac{1}{3+} + \frac{1}{4}$ as a regular fraction.
3. Prove the equalities $\frac{m}{n} = a_1 + \frac{p}{n} = a_1 + \frac{1}{\frac{n}{p}}$.
4. Use the method given in the text to write the fraction $\frac{251}{802}$ as a continued fraction.
5. Find out a reason to explain why the process will always end.