

Épreuve de section européenne

Compound events

If there are two independent events the respective probabilities of which are known, how to find the probability that both will happen ?

Suppose that the first event may happen in a ways and fail in b ways, all the cases being equally likely; and suppose that the second event may happen in a' ways and fail in b' ways, all these ways being equally likely. Each of the $a + b$ cases may be associated with each of the $a' + b'$ cases, to form $(a + b)(a' + b')$ compound cases all equally likely to occur.

In aa' of these, both events happen, in bb' of them, both fail, in ab' of them, the first happens and the second fails, and in $a'b$ of them, the first fails and the second happens.

Thus

$\frac{aa'}{(a + b)(a' + b')}$ is the chance that both events happen;

$\frac{bb'}{(a + b)(a' + b')}$ is the chance that both events fail;

$\frac{ab'}{(a + b)(a' + b')}$ is the chance that the first happens and the second fails;

$\frac{a'b}{(a + b)(a' + b')}$ is the chance that the first fails and the second happens.

Adapted from *Higher Algebra*, by Hall and Knight, 1964.

Questions

In the questions, we denote A the first event and A' the second event.

1. Use the letters a and b to express the probabilities of the event A and its complement, then the letters a' and b' to express the probabilities of the event A' and its complement.
2. Explain the sentence “Each of the $a + b$ cases may be associated with each of the $a' + b'$ cases, to form $(a + b)(a' + b')$ compound cases all equally likely to occur.”
3. Illustrate the situation with a probability tree, and deduce the 4 formulas given at the end of the document.
4. Supposing that it is 9 to 7 against a person A who is now 35 years of age living till he is 65, and 3 to 2 against a person B now 45 living till he is 75; find the chance that one at least of these persons will be alive 30 years hence.