Épreuve de section européenne

Balanced Ternary

We usually represent numbers in the base-10 positional system, where "one hundred twenty-one" is written 139, because it is equal to $1 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$. Using the same principle in a base-3 system, 1221 represents the decimal number $1 \times 3^3 + 2 \times 3^2 + 2 \times 3^1 + 1 \times 3^0$, which is equal to 52. Obviously, in the base-3 (also called ternary) system we need only three digits, namely 0, 1 and 2, and it has been proved that this system is almost always the most economical one, as far as computational efficiency is concerned.

In balanced ternary, each digit can be a negative unit (denoted N), a zero (0), or a positive unit (1). They are "balanced" because they are arranged symmetrically about zero.

As an example, the decimal number 19 is written $1N01$ in balanced ternary, and this numeral is interpreted as follows: $1 \times 3^3 - 1 \times 3^2 + 0 \times 3^1 + 1 \times 3^0$ or in other words $27 - 9 + 0 + 1$. Every number, both positive and negative, can be represented in this scheme, and each number has only one such representation. The balanced ternary counting sequence begins: 0, 1, $1N$, 10, 11, $1NN$. Going in the opposite direction, the first few negative numbers are N, $N1$, $N0$, NN. Note that negative values are easy to recognize because the leading digit is always negative.

What makes balanced ternary so pretty? It is a notation in which everything seems easy. Positive and negative numbers are united in one system, without the bother of separate sign bits. Arithmetic is nearly as simple as it is with binary numbers; in particular, the multiplication table is easily built. Addition and subtraction are essentially the same operation: Just negate one number and then add. Negation itself is also effortless: Change every N into a 1, and vice versa.

Inspired by Glusker, Hogan & Vass, "The ternary calculating machine of Thomas Fowler", *IEEE Annals of the History of Computing*, 27-3, 2005 and other sources.

Questions

1. Explain the balanced ternary counting sequence and continue writing it up to the 10th numeral. In the same manner, explain the negative number sequence and write it to the 8th negative numeral.
2. Write the balanced ternary number $1N10$ in base-10.
3. Write the decimal number 52 in balanced ternary.
4. Which property of balanced ternary is especially useful for money computations?
5. Build the addition and multiplication tables in balanced ternary numeration system (hint: check that $1 + 1$ is $1N$; $N + N$ is $N1$).
6. Use these tables to do the following operations: $1N01 + 11N0$; and $1N11 \times 1N0$.