Épreuve de section européenne

Logarithm tables

The point with logarithm is that when numbers are multiplied, their logarithms are added and when they are divided, the logarithms are subtracted. By hand, addition is much simpler than multiplication. The following is the process of multiplying two numbers using logarithms:

Look up their logarithms in tables.
Add the two logarithms.
Look up in antilogarithm tables.

The Scottish mathematician John Napier (1550-1617) discovered the logarithms, and Henry Briggs published Napiers's logarithms as tables and helped them gain acceptance among the scientific and academic communities. Laplace called logarithms an "admirable artifice which, by reducing to a few days the labor of many months, doubles the life of the astronomer, and spares him the errors and disgust inseparable from long calculations".

In these tables, the logarithm used is such that the logarithm of 10 equals 1 and is called common logarithm. For any positive number x we denote it $\log(x)$. We denote $\ln(x)$ the natural logarithm of x, for which $\ln(e) = 1$. Then, we have :

$$\log(x) = \frac{\ln(x)}{\ln(10)}. \quad (1)$$

x	$\log(x)$
1.789	0.25261
...	...
2.310	0.36366
...	...
4.133	0.61627
...	...
7.393	0.86882
7.394	0.86888
7.395	0.86894
7.396	0.86900

Here is an example to see the usefulness of the logarithm tables: let us suppose we want to compute the product $p = 1789 \times 413.3$ using a table giving the common logarithms for numbers from 1 to 10 with a step of 0.001. We first write the scientific notations: $1789 = 1.789 \times 10^3$ and $413.3 = 4.133 \times 10^2$. Then the properties of logarithms give:

$$\log(p) = \log(10^5) + \log(1.789) + \log(4.133). \quad (2)$$

Finally, we read the logarithm table on the right and we can write $\log(1.789) + \log(4.133) \approx \log(7.394)$.

Adapted from R. Solomon, *The little book of mathematical principles*, NH, 2008 and other sources.

Questions

1. The use of logarithm tables was common until the 1970s, but then declined. Nowadays these tables are much less useful. Explain why.
2. Explain why, according to Laplace, the artifice of logarithm "doubles the life of the astronomer".
3. Complete the computation of p by hand (without the help of a calculator).
4. (a) Write the properties of the function ln (natural logarithm) about the product $a \times b$ and the quotient $\frac{a}{b}$ of two positive numbers a and b.
   (b) Using definition (1), prove the function log (common logarithm) obeys the same rules.
5. Prove the affirmation (2).
6. Explain how we could find the result of the quotient $\frac{41.33}{17.89}$ using the difference of two logarithms.