Épreuve de section européenne

The amazing number 142857

A number beloved by mathematicians is $N = 142857$, the decimal period of the reciprocal of seven. Indeed, when we compute $\frac{1}{7}$, the result is $0.142857142857142857\ldots$ This number $\frac{1}{7}$ is the first decimal reciprocal to have maximum period, that is to say the length of this period is only 1 less than the number itself. In fact, for any prime number p, the period of $\frac{1}{p}$ is a divisor of $p-1$.

Multiplication by any number from 1 to 6 gives a cyclic permutation of the same numbers: $142857 \times 2 = 285714$, $142857 \times 3 = 428571$, $142857 \times 4 = 571428$, etc. Multiplication by seven also gives a strange result.

Multiplication by higher numbers produces the same pattern again, but with a slight difference. For instance, $142857 \times 12 = 1714284$, which becomes 714285 when an extra 1 is taken from the front and added to the units place.

Another example : multiplying 142857 by itself gives 20408122449. Separate this number into two groups of 6 digits from the right, add them, and you will find the initial number.

If we group the digits of 142857 in pairs, we find 99. We can also group the digits of any multiple of 142857 (with the exception of zero) in different ways, and find interesting results. For instance, with $142857 \times 361 = 51571337$, we have $51+571+377 = 999$ and $51+57+13+77 = 198$ which becomes 99.

Adapted from *Numbers*, David Wells, Penguin Books, 1997

Questions

1. What does “reciprocal” mean?
2. Compute 142857×5 and 142857×6 and check that each result is a cyclic permutation of 142857.
3. What is strange about the product of 142857 and 7?
4. Compute 142857×13 and comment on your result.
5. Explain the sentence “multiplying 142857 by itself...number”.
6. Compute 142857×77, group some digits in pairs and comment on your result.
7. Check that the reciprocal of 13 has a period which is a divisor of $13 - 1 = 12$. What about the reciprocal of 37?