Épreuve de section européenne

Pascal's triangle

Blaise Pascal

The set of numbers that form the famous Pascal's triangle were well-known before Blaise Pascal (1623-1662), a French mathematician and philosopher, also a genius of engineering and literature. These numbers originally came from Hindu studies of combinatorics and binomial numbers in Greek studies. But Pascal was actually the first one to develop and to organize all the information together for this triangle in a treatise, *Traité du triangle arithmétique* (1653).

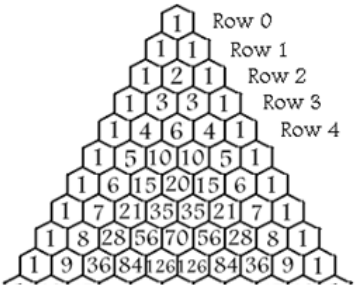
Here is the way to create Pascal's triangle. At the top of Pascal's Triangle is the number 1, which makes up row zero. The first row contains two 1's, both formed by adding the two numbers above them to the left and to the right, in this case 1 and 0 (all numbers outside the triangle are 0's). Do the same to create the second row: $0+1=1$; $1+1=2$; $1+0=1$. And the third row: $0+1=1$; $1+2=3$; $2+1=3$; $1+0=1$. In this way, the rows of the triangle go on infinitely.

This triangle has interesting properties.

First, the sum of the numbers in any row is equal to 2 to the n-th power, when n is the number of the row. For example: for row 3, $1+3+3+1=2^3$.

Secondly, if the second element in a row is a prime number, all the numbers in that row (excluding the 1's) are divisible by it.

Thirdly, if a row is made into a single number by using each element as a digit of the number (carrying over when an element itself has more than one digit), the number is equal to 11 to the n-th power, when n is the number of the row the multi-digit number was taken from. Indeed, $11^2=121$, $11^3=1331$, etc.



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Adapted from various sources.

Questions

1. Explain how to compute the numbers on row 10.
2. What number is equal to the sum of the numbers in a row? Try it with rows 1, 2, 3 and 4. Which row has a sum equal to 256?
3. Explain what is special when the second element in a row is a prime number. Give an example.
4. Explain how the triangle can be used to find the value of 11^4, without any calculator.
5. Explain the link between the numbers of this triangle, binomial expansions and combinatorics. Expand $(x+y)^7$ using this triangle.