

Épreuve de section européenne

The second derivative

In calculus, the second derivative f'' of a function f is the derivative of the derivative f' of f .

Roughly speaking, the second derivative measures how the rate of change of a quantity is itself changing. On the graph of a function, the second derivative corresponds to the curvature or concavity of the graph. The graph of a function with positive second derivative curves upwards, while the graph of a function with negative second derivative curves downwards. The second derivative of a quadratic function is constant.

For example, the second derivative of the position of a vehicle with respect to time is the instantaneous acceleration of the vehicle, or the rate at which the velocity of the vehicle is changing, whereas the first derivative is the rate at which the distance is changing, which is the instantaneous speed, or velocity of the car.

Let us consider two other examples :

1. A weather forecaster observing the atmospheric pressure p at time t may not be too concerned if p' is negative : pressure goes up and down all the time ! But if she also notices that p'' is [...], it may be time to issue a warning of severe weather.
2. Economists and manufacturers will observe that the number of households possessing a computer has been increasing for a long time. But to plan ahead they need to know whether this rate of increase is itself [...] (in which case they should increase production of models for first-time users) or [...] (in which case they might target existing customers to update to more sophisticated equipment). So the value of f'' affects strategic planning decisions.

Adapted from *Wikipedia* and various sources

Questions

1. What can you say about f' when f'' is positive ? And when f'' is negative ?
2. Draw two quick sketches to illustrate the fourth sentence of the text (“The graph. . . downwards”).
3. Prove that “the second derivative of a quadratic function is constant”.
4. Complete the blanks ([...]) in the 2 examples, and comment on them.
5. Explain the word “instantaneous”, which appears twice at the end of the first paragraph of the text, using the definition of the derivative.