Épreuve de section européenne

Euclid's solution of some quadratic equations

- *Brief history of the quadratic equation*

  It is often claimed that the Babylonians (about 1600 BC) were the first to solve quadratic equations. This is an oversimplification, for the Babylonians had no notion of "equation". What they did develop was an algorithmic approach for solving problems which, in our terminology, would give rise to a quadratic equation. However all Babylonian problems had answers which were positive (more accurately unsigned) quantities since the usual answer was a length.

  In about 300 BC Euclid developed a geometrical approach which amounted to finding a length which, in our notation, was the root[1] of a quadratic equation. Although later mathematicians used it to actually solve quadratic equations, Euclid had no notion of equation, coefficients etc. but worked with purely geometrical quantities.

  Hindu mathematicians took the Babylonian methods further so that Brahmagupta (598-665 AD) gave a method which admits negative quantities. He also used abbreviations for the unknown, usually the initial letter of a colour was used, and sometimes several different unknowns occur in a single problem.

  The final, complete solution as we know it today came around 1100 AD, by another Hindu mathematician called Baskhara. Baskhara was the first to recognise that any positive number has two square roots.

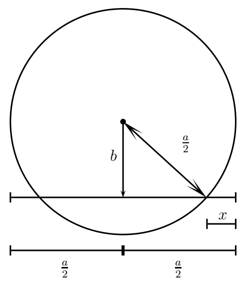
- *Euclid's method*

  Euclid's method for solving quadratic equations only applies to equations of the following type :

  $$x^2 - ax + b^2 = 0 \qquad \text{with } a > 0 \text{ and } b > 0. \tag{1}$$

  It goes as follows, the picture on the right helping to follow the different steps :

  1. Draw a line of length a.
  2. Draw a line of length b perpendicular to the previous one, from its midpoint.
  3. Draw a circle centred at the end of this line, with radius $\frac{a}{2}$.
  4. The length denoted x gives the solution of the equation $(a-x)x = b^2$.



From The MacTutor History of Mathematics.

Questions

1. Which mathematician mentioned in the text was one of the first to use a letter to represent the unknown?
2. Give an equation with at least one negative root ?
3. Check that the expression in point 4 of Euclid's method is equivalent to equation (1).
4. At which condition on a and b does equation (1) admit 2 solutions ?
5. Find out the solutions of $x^2 - 5x + 4 = 0$ with the modern method. Does Euclid's method apply in this case ?
6. Draw the figure corresponding to the equation above. Measure the length called x in Euclid's method and check that it is indeed a solution of $x^2 - 5x + 4 = 0$.
7. Prove that Euclid's method is valid.

[1] root : solution