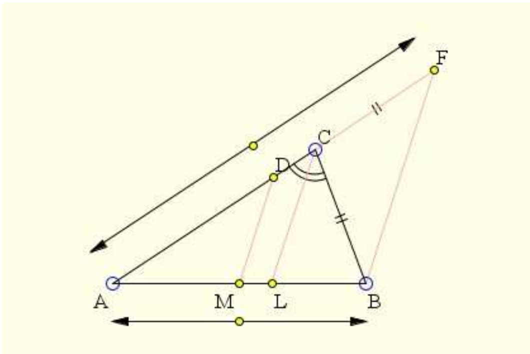
Épreuve de section européenne

Cleavers

Let ABC be any triangle. Let AC be extended to F so that $CF = CB$, and M be the midpoint of AB. Draw a parallel line to BF from M, intersecting line AC at D. MD will have half the perimeter of ABC on each side of it.

Let us call a perimeter-bisecting segment like MD a *cleaver* when it issues from the midpoint of a side.



A remarkable fact can be proved:

The three cleavers of a triangle always meet in a *cleavance-centre*.

A nice proof of this can be based on the engaging property that each cleaver is parallel to an angle bisector of the triangle ABC: let CL be the angle bisector of angle C. MD is parallel to FB, therefore $\angle ADM = \angle CFB = \angle CBF$ and $\angle ACL = \angle LCB = \angle CBF$. So, the cleaver MD is parallel to the angle bisector CL of angle C. (We note in passing that this gives a nice construction for a cleaver.)

The medial triangle MNP: Triangle MNP, determined by the feet of the medians, that is, the midpoints of the sides, is called the medial triangle of $\triangle ABC$. The cleavance-centre is simply the *incentre* S of the medial triangle (remember that the incentre of a triangle is the centre of the incircle and is accordingly the point of intersection of the three angle bisectors).

Adapted from Episodes of *Nineteenth and Twentieth Century Euclidian Geometry*, by Ross Honsberger

Questions

1. Translate algebraically the sentence "MD will have half the perimeter of ABC on each side of it" then prove that it is true.

2. (a) Why is MD parallel to FB?
   (b) Explain the inequalities $\angle ADM = \angle CFB = \angle CBF$ and $\angle ACL = \angle LCB = \angle CBF$.
   (c) Hence deduce that the line MD is parallel to the angle bisector CL.

3. Draw a triangle ABC.
   (a) Describe the construction of the cleaver DM mentioned at the end of the third paragraph.
   (b) Use the last paragraph of the text to explain a geometrical construction of the cleavance centre S.