Épreuve de section européenne

Converse and contrapositive

Let's consider the statement "if P then Q". It's important not to confuse the converse "if Q then P" and the contrapositive "if not Q then not P" !

For instance, if the statement is "all red objects are colored", it can be expressed as "if an object is red, then it is colored". So the converse is "if an object is colored, then it is red" whereas the contrapositive is "if an object is not colored, then it is not red". The same reasoning can be used with the statement "all quadrilaterals have four sides".

Be careful with these two logical sentences ! Notice that if a statement is true, the contrapositive must be true, but the converse can be true or not. If you want a statement and its converse to be both true or false, they have to be equivalent. For instance, "$x^n = 1$ ($n \in \mathbf{N}$)" and "$x = 1$" are not always equivalent statements.

Sometimes it is easier to prove a result in the contrapositive form rather than in the original conditional form : for instance "if the square of a natural number is not odd, then this number is not odd". It can also be used for everyday life like "if the ground is not wet, then it is not raining".

Some statements become more difficult to put in the contrapositive form when they involve several connectives (and, or…): for instance "If x is odd and y is even then xy is even" or "If x is not less than zero or not greater than one, then x^2 is less than 1".

Adapted from *www.math.csusb.edu* and various sources

Questions

1. Give the converse and the contrapositive of "all quadrilaterals have four sides".
2. Explain why "$x^n = 1$ ($n \in \mathbf{N}$)" and "$x = 1$" are not always equivalent statements.
3. Using the contrapositive, prove that the statement "if the square of a natural number is odd, then this number is odd" is true.
4. Try to write the contrapositive of "If x is odd and y is even then xy is even".
5. Is the last statement of the text true or false ? Explain.